Math 2351

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Good luck for all

Section 1.6

liner Modles:- Cost تكلفة, Revenue عائد, profit الربح

and Brak Evenpoint.

Review :-

linear function so

$f(x) = y = mx + b$

ميلان slope

y-interspt → $(x=0, b)$

x-interspet → $(y=0, x)$.

$* \; m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

زوج مرتب

point – slope form .

Given a point (x_1, y_1) and a slope m the equahuinis .

$y - y_1 = m(x - x_1)$. → معادلة الخط المستقيم

Cost = y → يعني الكوست هي نفسها y وتوزلها برمز C

عندما يكون عندي زوجين مرتبين بأخذ زوج مرتب واحد وبعوضه في المعادلة .

Ex:- $(5,7)$ $(10,-2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{10-5} = \frac{-9}{5}$ → slope.

عندما يكون (slope) سالب يكون الخط المستقيم متناقص (decreasing) وعندما يكون (slope) بالموجب يكون الخط المستقيم متزايد (increasing)

$y - y_1 = m(x - x_1)$. ← أوجد معادلة الخط المستقيم

$y - 7 = -\frac{9}{5}(x - 5)$.

$y - 7 = -\frac{9}{5}x + 9$

$+7 \qquad\qquad +7$

$y = \frac{-9}{5}x + 16$

في السكشن الاول رح نتعرف على عدة اقترانات وكلهم على معادلة الخط المستقيم فرح نستخدم معادلة الخط المستقيم لكن رح يتغير مطلب السؤال في كل معادلة حسب نوع الاقتران ..

$y = mx + b$ → هذا الشكل العام للمعادلة الخطية.

C = Cost	$C = mx + b$
R = Revenue.	$R = mx + b$
P = Profit.	$P = mx + b$.
B = Bark Even....	$B = mx + b$.

في كل مرة رح يتغير المطلوب حسب نوع الاقتران .

عدد الوحدات

* Total cost : C(X) :- التكاليف الكلية

X: the number of items produced :- ترمز (X) إلى عدد الوحدات ولكن لا يمكن ~~عدد~~ ~~ولكن أن تكون عدد~~
$0 < x < \infty$ أن تكون سالب لأنه لا يوجد وحدات بالسالب قيمها (0 ، ∞)

(تعتمد على عدد الوحدات)

C(X) = Variable cost + Fiexd cost.

التكاليف الكلية = التكاليف المتغيرة + التكاليف الثابتة ⟸) التكاليف الثابتة حتى لو أنا ما أنتجت لازم أدفعها

C(X) = VC + FC

* مثال :- يوجد عندي شركة سيارة بدي أنتج 4 إطارات (وحدات) للسيارات كل إطار بكلفني 5$ (سعر التكلفة) ويوجد عندي التكاليف الثابتة بتساوي 100000$ كم التكلف الكلية :-

C(X) = VC + FC — 4 → عدد الوحدات

C(X) = (4*5) + 100000 — 5 → سعر التكلفة لكل وحدة.

= 20 + 100000 — VC = 4.5 = 20

C(X) = 100020.

VC :- The variable cost is the cost per unite (slope (m)) multipled by the total number of units pradicet. تكلفة كل وحدة في (ضرب) عدد الوحدات.

VC = (cost per unit (تكلفة الوحدات)) . (number of units (عدد الوحدات)).
= m . X

VC = mX → m = cost per unit / X = number of units.

* ملاحظة مهمة :-

* Total cost

C(X) = VC + FC
C(X) = mX + b
إذا كان (cost) معادلة خطية فإن :-
m = slope = marginal cost ($\overline{MC}$)] تكلفة إنتاج وحدة وحدة فقط.

b = FC = C(0) ⟸ التكاليف الثابتة هي نفسها التكلفة عند الصفر (الانتاج)
عندما لا يكون الاقتران خطي وبدي قيمة (FC) بعوض محل X=0
C(X) = m.0 + b
C(X) = b (فتظهر عندي قيمة (B))

$\overline{MC}$:- the cost of producing one additional unit at any level of prodachion.

Example 8:
The Total Cost for a product is

$$\overset{mX}{C(X) = 10X} \overset{+}{+} \overset{b}{\underline{200}}$$

$FC = \underline{C(0)} = 200$

$C(0) = 10 \cdot 0 + 200$

$F(C) = C(0) = 200$

1- What is marginal Cost, and what does it men?

$\overline{MC} = m = 10\$$

mean $\overline{MC}$ = the cost of producing one additinal unite at any level of prodachion.

* تكلفة انتاج وحدة إضافية واحدة = مثلاً عندي X = 5 ← وضفت عليها وحدة واحدة صارو 6

كم مقدار الزيادة =

$C(5) = 10 \cdot 5 + 200$

$C(5) = 250$

$C(6) = 10 \cdot 6 + 200$

$C(6) = 260$

← مقدار الزيادة زادت بمقدار 10 وحدات . $\overline{MC}$

$C(21) - C(20) = \underline{10} \Rightarrow \overline{MC}$

2. Find the Fiexd Cost? = 200

3. What is the cost of producing 100 units? $\Rightarrow X = 100$

$C(X) = 10X + 200$

$C(100) = (10 \cdot 100) + 200$

$C(100) = 1200$

4. What is the cost of producing one more (مرة أضيف عليها وحدة) unite if 100 unites are carrently being product?

$C(100) = 10 \cdot 100 + 200 = 1200$

$C(101) = 10 \cdot 101 + 200 = 1210$ → الفرق بينهم يساوي 10 $

$\overline{MC} = 10\$$

5. Find $C(101) - C(100)$? $= 1210 - 1200 = 10\$$

$C(X)$:- التكاليف ← بالموجب
$R(X)$:- العائد ← بالموجب
$P(X)$:- الربح
يعتمدو على بعضهم البعض.

(profit) + ← → − (Loss)

$P(X) \Rightarrow$ له علاقة بـ $C(X)$ و $R(X)$

* Total Revene :- $R(X)$:- العائد.

العائد :- يعني أنا أنتجت وحدات وبدي أبيعهم فبتم بيعهم في السوق وذلك يصبح عندي مصدر دخل من البيع ← فبالتالي العائد يعتمد على السعر.

X = number of unites عدد الوحدات

مثلاً :- أنا عندي 10 وحدات بدي أبيع كل وحدة منهم بـ 5 كم $R(X)$ عندي :- $10 \cdot 5 = 50\$$

P = price per unite سعر كل وحدة ← سعر يكون ثابت إذا خطية.

$$R(X) = \underset{\text{سعر الوحدة}}{P} \cdot \underset{\text{عدد الوحدات}}{X}$$

* احتمال ما يعطيني قيمة X ← عادي ببقيها مجهول وكمان قيمة $B = 0$ ← إذا ما أعطاني ياها وسعر الوحدة = 5 كون معادلة خطية =

$$R(X) = 5X + 0$$

$\hookrightarrow$ $R(X) = mX + b$ = على شكل المعادلة الخطية

$R(X) = P \cdot X$ ← معادلة ثابتة

$$\text{Slope} = P = MR$$

EX ⇒ $R(X) = 150X \Rightarrow MR = 150$

P = Selling price = $\overline{MR}$ (marginal revenue :- the revenue of produceing and selling one additianal unite at any level of production). (revene.) لأنه عائد بيع

$$R(X) = 150X$$

مثال :- عندما أنتج 30 وحدة (أنتج وأبيع 30) وعندما أنتج الوحدة رقم (31) كم رح يزيد العائد عندي ⇐ طبعاً رح يزيد عندي بمقدار 150

$R(30) = 150 \cdot 30 = 4500$
$R(31) = 150 \cdot 31 = 4650$
زيادة عندي بمقدار 150 $ كل حول.

Example :-

A Company sells its products at 10$ per unite.

1. Find the Revene function:-

$R(X) = P \cdot X$
$R(X) = 10X$

2. Find the marginal revenue and interpet youre answer? → وضح إجابتك لماذا.

$\overline{MR} = 10$, the revenue of producing and selling one addicinal unite is 10$ at any level of production.

*Total profit :: $P(X)$:- الربح

$P(X) = R(X) - C(X)$.

الربح = العائد - التكلف

لذلك يمكن أن ينتج معي أجابات بسالب وإجابات بالموجب عندما :-

← يكون الجواب بالموجب يكون العائد أكبر من التكلفة ويكون عندي ربح.

← ويكون الجواب بسالب عندما يكون العائد أقل من التكلفة ويكون عندي في هذه الحالة خسارة.

if ($P(x)$ is liner funchon.

slope = MP = marginal profit: the profit of produaing and selling one addional unit at any level of production.

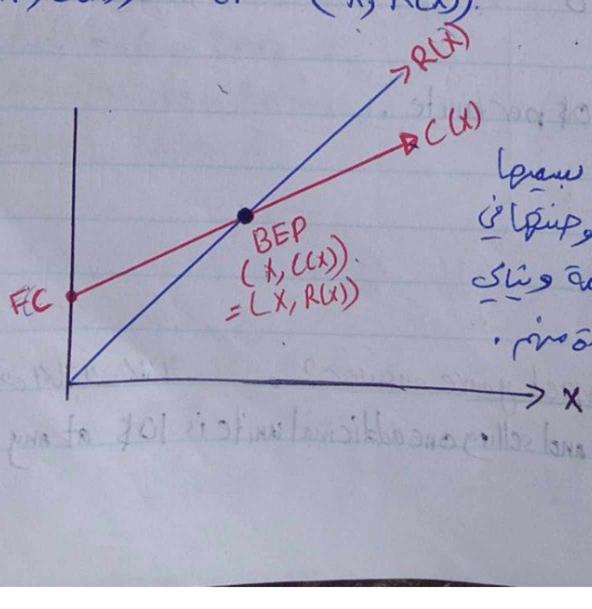
هو عبارة عن الربح من خلال إنتاج وبيع وحدة إضافية واحدة فقط.

$P(X)$:
- → positive → profit ⇒ إذا كان عندي العائد أكثر التكلف يكون عندي ربح
- → negative → loss ⇒ إذا كان عندي التكلف أكثر من العائد يكون عندي خسارة
- → = 0 → BEP (Brak Evenpoint) إذا كان عندي العائد والتكلف يساوي بعض [neither profit nor loss] يكون عندي لا خسارة ولا ربح.

mean of point :- (x , y) تعني زوج مرتب

BEP → $P(X) = 0$ or $C(X) = R(X)$.
$(X, C(X))$ or $(X, R(X))$.

واحد منهم تكفي لأنو الربح عندما يساوي صفر يعني لا يوجد عندي خسارة ولا ربح يعني التكلفة والعائد نفس المقدار.



* النقطة إلي بيتقاطع عندها R و C بسميها $(X, R(X))$ / $(X, C(X))$ قيمة X عوضتها في R أو C رح يطلع عندي نفس القيمة وبالتالي يطلع عندي زوج مرتب بأخذ واحدة منهم.

* Example :- question 11 → page 113.

A company charhing its profit noties that the relation between the number of units sold, x, and the profit $p(x)$, is liner. if 200 unite sold results in $3100 profit and 250 unites sold results in $6000. write the profit function? معادلة خطية (liner)
Find $\overline{MP}$?

$p(x) = ax + b$ معادلة الاقتران الخطي بهذا الشكل

⇒ :- if P is liner P → profit function.
200 unites → profit = 3100 ⇒ $P(200) = 3100$
250 unites → profit = 6000 ⇒ $P(250) = 6000$

* write the profit function, Find the marginal profit ?
(200, 3100) لازم أكتبهم على شكل أزواج مرتبة تصير عندي :-
(250, 6000) هنا عندي زوجين مرتبين من خلالهم بقدر أحل :-

1- slope = ??

$$m = \frac{6000-3100}{250-200} = 58$$

* يوجد عندي طريقتين للحل إما عن طريقة المعادلة الخطية أو عن طريق slope وزوج مرتب واحد.
m = 58 / (200, 3100).

1- $P(x) = mx + b$
$P(200) = (58 \cdot 200) + b$
$3100 = 11600 + b$
$-11600 \quad \cancel{-11600}$
$b = -8500$
$P(x) = 58x - 8500$
* طريقة الأولى للنظ فقط.

2- $y - y_1 = m(x - x_1)$
$y - 3100 = 58(x - 200)$
$y - \cancel{3100} = 58x - 11600$
$\cancel{+3100} \quad +3100$
$y = 58x - 8500$
↓
$P(x) = 58x - 8500$
* طريقة الثانية أسهل من الأولى

* وعلى أي طريقة أو أي قانون من إلى الفوق بطلع عندي نفس الجواب في نهاية.
وحتى كمان إذا استخدمت أي نقطة من زوجين المرتبين.

* Example 8:

A company sell its products at $25 per unite. The fixed cost is $1800 and the cost per unit is $15. Find the following:

⇒ pries per unite = 25 / fiexd cost = $C(0) = 1800$ / cost per unit = $m = 15$

الحل :-

1- Fined the cost Function ?

$C(x) = mx + b$

$C(x) = 15x + 1800$

2- Find the revenae Function ?

$R(x) = p \cdot x$

$R(x) = 25x$

3- Find the profit Function ?

$P(x) = R(x) - C(x)$.

$P(x) = 25x - (15x + 1800)$

$P(x) = 25x - 15x - 1800$

$P(x) = 10x - 1800$

4- Find $P(201) - P(200)$?

$P(201) - P(200) =$

$((10 \cdot 201) - 1800) - ((10 \cdot 200) - 1800) =$

$210 - 200 = 10$ → marginal profit

5- Find the BEP ?

$P(x) = 10x - 1800 = 0$

$10x \cancel{- 1800} = 0$
$\cancel{+1800} \quad +1800$

$\frac{\cancel{10}x}{\cancel{10}} = \frac{1800}{10} \Rightarrow x = 180$

BEP :-

$(180, C(180)) = (180, R(180))$

$(180, 4500) = (180, 4500)$.

*** Example :**

1)- The total cost of producing 20 units of a product is 1880\$ and the cost of producing 25 units is \$1950. The total revenue of selling 120 units of same product is \$2160. Assume linear cost and revenue models.

~~A- Find~~

A- Find the cost, the revenue and the profit functions?

$C(20) = 1880 \rightarrow (20, 1880)$

$C(25) = 1950 \rightarrow (25, 1950)$

$R(120) = 2160$

1- $C(x) = mx + b$ $\quad (m = 14 / (20, 1880))$

$m = \dfrac{1950 - 1880}{25 - 20} = \underline{\underline{14}}$ طريقة ثانية $\Rightarrow y - y_1 = m(x - x_1)$

$y - 1880 = 14(x - 20)$

$y \cancel{-1880} = 14x - 280$
$\quad \cancel{+1880} \qquad + 1880$

$y = 14x + 1600$

$C(x) = 14x + 1600$

$C(x) = 14x + b$

$C(20) = 14 \cdot 20 + b$

$1880 = \cancel{280} + b$
$-280 \quad \cancel{-280}$

$b = 1600$

$C(x) = 14x + 1600$

2- $R(x) = p \cdot x$ $\quad \Rightarrow R(120) = 2160$.

$R(120) = p \cdot 120$

$\dfrac{2160}{120} = \dfrac{\cancel{120}p}{\cancel{120}}$

$p = 18$

$R(x) = 18x$.

3- $P(x) = R(x) - C(x)$

$= 18x - (14x + 1600)$

$P(x) = 4x - 1600$

تكملة السؤال $\Longrightarrow$

*Complet Example :   تكملة المثال السابق.

$\Rightarrow P(X) = 4X - 1600$

B- Find the profit (or loss) when 200 unite are produced and sold.

$P(X) = 4X - 1600$

$P(200) = (4 \cdot 200) - 1600$

$P(200) = 800 - 1600$

$P(200) = -800$ $\therefore$ loss.

C- Find the profit (or loss) when 400 unite are produced and sold.

$P(X) = 4X - 1600$

$P(400) = (4 \cdot 400) - 1600$

$P(400) = 1600 - 1600$

$P(400) = 0 \rightarrow \text{BEP} \rightarrow$ يعني لا يوجد عندي ربح أو خسارة

D- Find the Break-even point?

$P(X) = 4X - 1600$

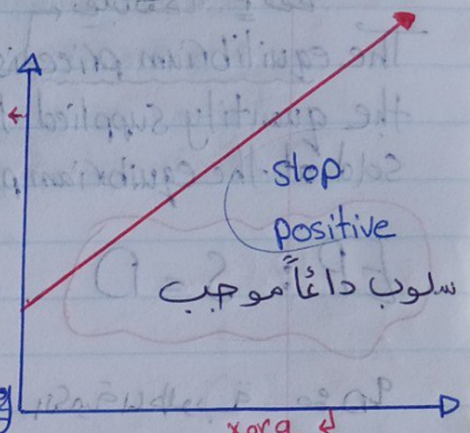
$0 = 4X \cancel{- 1600}$
$+1600 \quad \cancel{+1600}$

$\frac{1600}{4} = \frac{4X}{4} \Rightarrow (X = 400)$

$\text{BEP} \Rightarrow (X, C(X))$ or $(X, R(X))$.

$X = 400$	$X = 400$
$C(400) = (14 \cdot 400) + 1600$	$R(400) = 18 \cdot 400$
$C(400) = 5600 + 1600$	$R(400) = 7200$
$C(400) = 7200$	$(400, 7200)$
$(400, 7200)$	

* linear mothdes :- supply العرض , Demand الطلب , and Equilibram point نقطة التوازن



Supply :- A supply curve describes the relation between the quantity supplied and the selling price.

q or x : (the quantity supplied) the maximum amount that producers are willing to supply at agiven price.

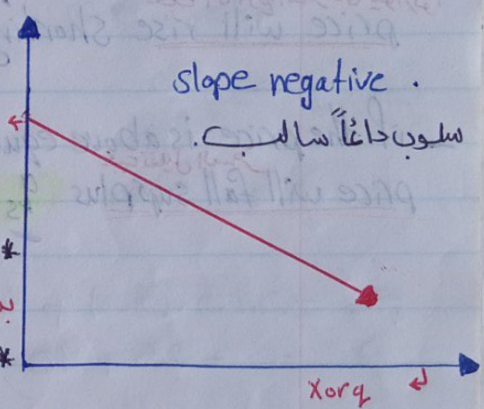
The law of supply : as prise increases, the corresponding quantity for sale will also increase.

* العلاقة بين السعر والكمية (علاقة طردية) يعني اذا زاد السعر زادت الكمية $p\uparrow \to q_s\uparrow$

* ودائماً السلوب لـ (supply) دائماً موجب .

نفس المعادلة فقط الرموز تختلف
$$\left[\begin{array}{l} p = 2q + 1 \\ y = 2x + 1 \end{array}\right.$$

* Demand :- quantity Demand q : the maximum amount that Consumer can be expected to buy at agiven price.



The Demand law :- as price increase, the corresponding [quantity demand will decrease.]

* العلاقة بين السعر والكمية (علاقة عكسية) يعني كل ما زاد السعر الكمية المطلوبة بتقل :- $p\uparrow \to q_D\downarrow$

* ودائماً سلوب لـ (Demand) دائماً سالب .

: EX :-

* فرق المعادلات الاتية من Demand or Supply :Q

1- $p = 2q + 10$ ⟶ Supply :- Becouse the slope positve.

بسبب أن السلوب موجب عندي يكون السلوب موجب دائماً يكون عرض ← (supply)

2- $p = -2q + 10$ → Demand :- Becouse the slope is negative.

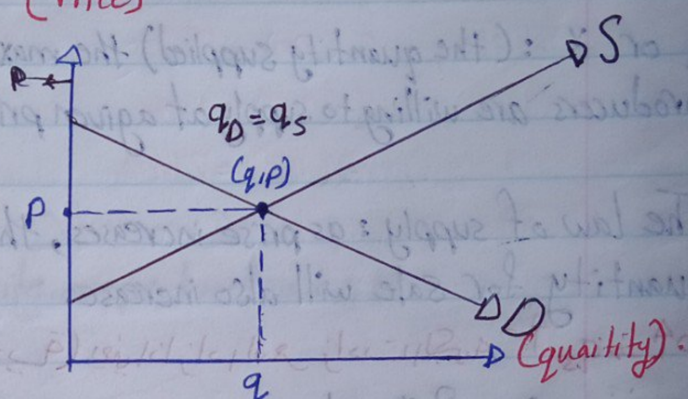
بسبب أن السلوب سالب عندي يكون السلوب سالب دائماً يكون طلب ← (Demand.)

* Equilibrium point :- (q, p) :- نقطة التوازن

يطلب نقطة P فقط.

The equilibrium price is the price at which the quantity Demand equals the quantity supplied the equilibrium price (سعر التوازن) is the quantity bought and sold at the equibriam price.

$$EP: S = D$$



q_D :- الكمية المطلوبة

q_S :- الكمية المعروضة

* Notes :-

- if the price is belaw equilibriam there will be shorting (عجز) and the price will rise (يجب ان أرفع السعر حتى يتوازنوا) shorting $q_D > q_S$ * إذا زادت الكمية المطلوبة عن الكمية المعروضة يصبح هناك عجز
- if the price is above equilibriam there will be a surplus (فائض) and the price will fall (يجب تقليل السعر) surplus $q_S > q_D$ * إذا زادت الكمية المعروضة عن الكمية المطلوبة يصبح هناك فائض

Q:-1

* Consider the following linear supply and demand relationshipe:- معادلات خطية
Producers will supply 1000 units when the selling price is \$20 per unit, and 1500 units when the price is \$25 per unit. Consumerse will demand 500 units when the selling price is \$15 per unit but the demand will decreace by 100 units if the price incrases by \$5. Both supply and demand funiction are linear. Determine the supply function, the demand function and the equilibrium point (using point – slope equations).

$p \to y$
$q \to x$

Supple :- (1000, 20) / (1500, 25)
Demand :- (500, 15) / (500−100, 15+5) ⟹ (400, 20)

حسب معطيات السؤال ينقص بمقدار 100 ويزاد بمقدار 5 .

* Supple ⟹

$$1-\ m = \frac{25-20}{1500-1000} = \frac{5}{500} = \frac{1}{100} = 0{,}01$$

slop = 0,01 / (1000, 20).

$$y - y_1 = m(x - x_1).$$
$$y - 20 = 0{,}01(x - 1000)$$
$$y - 20 = 0{,}01x - 10$$
$$+20 \qquad\qquad +20$$
$$y = 0{,}01x + 10$$

$$P = 0{,}01q + 10 \to \text{Supply liner.}$$

* Demand ⟹

$$m = \frac{20-15}{400-500} = \frac{5}{-100} = -0{,}05$$

Slop = −0,05 / (500, 15)

$$y - y_1 = m(x - x_1).$$
$$y - 15 = -0{,}05(x - 500)$$
$$y - 15 = -0{,}05x + 25$$
$$+15 \qquad\qquad +15$$
$$y = -0{,}05x + 40$$

$$P = -0{,}05q + 40 \to \text{Demand liner}$$

تكملة الحل ⟹

Q 1 :-

$\rightarrow$ تكملت الحل .

$\Rightarrow$ $S: P = 0.01q + 10$

$D: P = -0.05q + 40$

* EP :-

$S = D$

$0.01q + 10 = \cancel{-0.05q} + 40$
$+ 0.05q \qquad \cancel{+0.05q}$

$0.06q \cancel{+10} = 40$
$\cancel{-10} \qquad -10$

$\dfrac{\cancel{0.06}q}{\cancel{0.06}} = \dfrac{30}{0.06}$

$q = 500$

$P = -0.05 \cdot (500) + 40$
$P = -25 + 40$

$P = 15$

$(500, 15)$.

معادلة D حسب ترتيب السؤال

Q2 :-

بترتيب

The demand and supply functions are given, respectively: $\boxed{2p = 150 - q}$, and $\boxed{10p = 150 + q}$. Find the quantity demanded and the quantity suppled at each level of price. Determine whether there is a shortage or surpluses at.

معادلة S حسب ترتيب السؤال

a. $p = 20\$$ b. $p = 30\$$ c. $p = 25\$$.

$q_D > q_s$ (shortage)

$q_s > q_D$ (surpluses)

$q_s = q_D$ (EP).

$D = 2p = 150 - q$ / $S : 10p = 150 + q$.

a. $p = 20\$$.

$D = 20.2 = 150 - q$

$40 = 150 - q$

$-150 \quad -150$

$-q = -110$

$q_D = 110$

$S = 10.20 = 150 + q$

$200 = 150 + q$

$-150 \quad -150$

$q = 50$

$q_s = 50$

$110 > 50$

$q_D > q_s \rightarrow$ (shortage.)

b. $p = 30\$$.

$D = 30.2 = 150 - q$

$60 = 150 - q$

$-150 \quad -150$

$q = 90$

$q_D = 90$

$S = 10.30 = 150 + q$

$300 = 150 + q$

$-150 \quad -150$

$q = 150$

$q_s = 150$

$90 < 150$

$q_D < q_s \rightarrow$ (surpluses).

c. $p = 25\$$.

$D = 25.2 = 150 - q$

$50 = 150 - q$

$-150 \quad -150$

$-100 = -q$

$q_D = 100$

$S = 25.10 = 150 + q$.

$250 = 150 + q$.

$-150 \quad -150$

$q = 100$

$q_s = 100$

$100 = 100$

$q_D = q_s \rightarrow$ (Equilibrumpoint).

Q 3 :-

The demand for a certain commodity is given by $5p + 2x = 200$ and the supply is given by $5p = 4x + 50$.

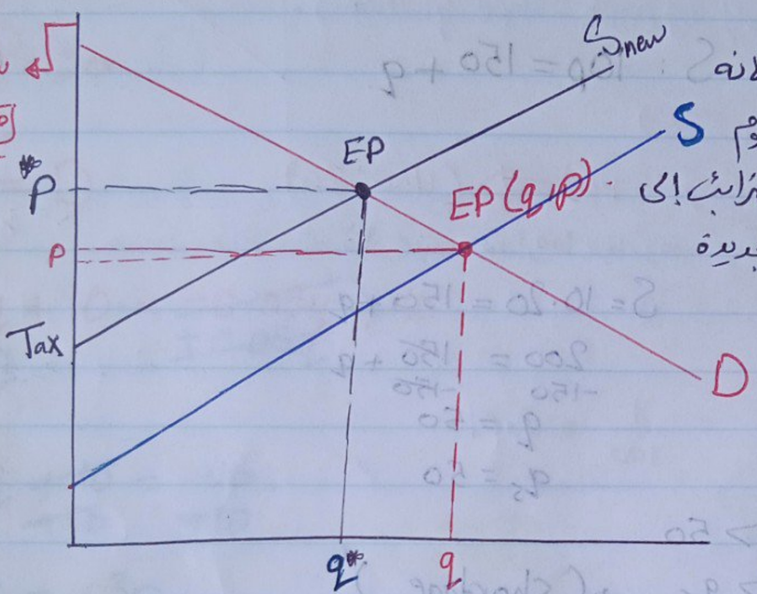
a). Find the equilibrium price and quantity.

يؤدي على زيادة الاسعار وتغيير يكون على (S) م

b) Find the equilibrium price and quantity after a tax of 6 per unite is imposed.

Tax شرح ك

مهم



Tax ← تأثير يكون على (supply) لانه ضرائب دائماً بتكون على البائع والبائع تقوم برفع على الاسعار حتى يتناسب مع الضرائب إلى انفرضت عليه (وتصبح نقطة التوازن الجديدة الكمية قليلة والسعر مرتفع).

a)- $S = D$

حل المعادلات بالحذف.

$D = \cancel{5p} + 2X = 200$

$S = \cancel{5p} - 4X = 50$

$\frac{6X}{6} = \frac{150}{6}$

$\boxed{X = 25}$

. $25 = X$ تعويض

$5p + 2 \cdot 25 = 200$

$5p + 50 = 200$
$\quad\;\; -50 \quad -50$

$\frac{5p}{5} = \frac{150}{5}$

$p = 30$

$EP \to (25, 30)$

$25 > 20$

$30 < 32$

b). $S \Rightarrow \frac{5p}{5} = \frac{4X + 50}{5}$ — إضافة الضريبة :-

$\Rightarrow p = \frac{4}{5}X + 10$

$S_{new} \Rightarrow p = \frac{4}{5}X + 10 + 6$

$S \Rightarrow p = \frac{4}{5}X + 16$

$D \Rightarrow 5p + 2X = 200$

$EP \Rightarrow S_{new} = D$. — اكمل تعويض

$5(\frac{4}{5}X + 16) + 2X = 200$

$4X + 80 + 2X = 200$

$6X = 200 - 80$

$\frac{6X}{6} = \frac{120}{6} \Rightarrow X = 20$

$p = \frac{4}{5} \cdot 20 + 16$

$p = 32$

$EP_{new} = (20, 32) \to$

(q^*, p^*)

تأثير الضريبة على الحل .
لازم تكون الكمية أقل و السعر أعلى من الحل قبل الضريبة .

Section 2.3
Quadratic Models.

Review: Quadratic Functions (parabola).

$$y = f(x) = ax^2 + bx + C \quad , a \neq 0$$

Graph:-

1. $a > 0$ upward ∪ → optimal minimum (أقل قيمة.)
   $a < 0$ downward ∩ → maximum (أعلى قيمة.)

2. vertex $\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$ → بتحددلي

3. Zeros (x – intercept)

$y = 0$

$ax^2 + bx + C = 0$.
- → Factoring → ( ).( ) تحليل الاقواس
- → quadratic Formula → القانون العام.

$$x = \frac{-b \pm \sqrt{d}}{2a}$$

$$d = b^2 - 4ac$$

$d > 0$ → two solutions → يوجد حلين (قيمتين).

$d < 0$ ⇒ no. solution. → لا يوجد حل.

$d = 0$ ⇒ one solution → $x = \frac{-b}{2a}$. → يوجد قيمة واحدة.

Example:-

The profit of selling X units of a product is given by $\boxed{P(X) = 12X - 0,1X^2}$. What is the maximum profit and how many unites should be sold in order to earn this maximum profit)

$$P(X) = 12X - 0.1X^2$$

$a = -0.1 < 0$ (maximum) ∩

maximum profit $f\left(\frac{-b}{2a}\right)$. vertax $X \Rightarrow \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

$a = -0.1, \; b = 12, \; C = 0$

$X = \frac{-b}{2a} \Rightarrow \frac{-12}{2(-0.1)} = \frac{+12}{+0.2} = 60$ number of unite that given the maximum profit.

$P(X) = 12X - 0.1X^2$

$P(60) = 12 \cdot 60 - 0.1(60)^2$

$720 - 360 = 360$

maximum profit $= \underline{360}$

Q1 :-

Let $\vec{p} = 25 - 0.01x$ and $C(x) = 2x + 9000$ be the price-demand equation and cost function, respectively, for the manufacturer of unmbrellas.

a). Find the maximum profit ?

$* \; C(x) = 2x + 9000$

$* \; R(x) = p \cdot x$ Demand دا عاً بتكون تاخن

$R(x) = (25 - 0.01x) \cdot x$

$R(x) = 25x - 0.01x^2$

$* \; P(x) = R(x) - C(x)$.

$P(x) = 25x - 0.01x^2 - (2x + 9000)$.

$P(x) = 25x - 0.01x^2 - 2x - 9000$

$P(x) = -0.01x^2 + 23x - 9000$

$\rightarrow$ maximum $x = \frac{-b}{2a}$

$a = -0.01 \quad b = 23 \quad c = -9000$

$x = \frac{-23}{2(-0.01)} = \frac{23}{0.02} = 1150$

$P(1150) = -0.01(1150)^2 + 23(1150) - 9000$

$P(1150) = -13225 + 26450 - 9000$

$P(1150) = 4225$

maximum profit $= 4225$.

b). Find the maximum revenue ?

$* \; R(x) = 25x - 0.01x^2$

$a = -0.01 \quad b = 25 \quad c = 0$

$x = \frac{-b}{2a} = \frac{-25}{-2(0.01)} = \frac{25}{0.02} = 1250$

$R(1250) = 25(1250) - 0.01(1250)^2$

$R(1250) = 31250 - 15625$

$R(1250) = 15625$

maximum revene $= 15625$.

$\Rightarrow$ تكملة السؤال .

Q1 :- ⟸ يتبع

c)- Find the number of unites that must be sold to guarantee no loss لا يوجد خسارة

$P(X) = 0$

$\frac{-0.01X^2}{-0.01} + \frac{23X}{-0.01} - \frac{9000}{-0.01} = 0$

$X^2 - 2300X + 900000 = 0$

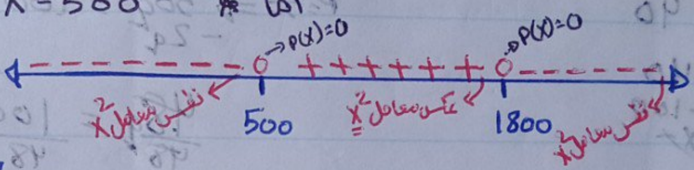
$(X - 1800) \quad (X - 500) = 0$

$X = 1800$ or $X = 500$ اما

✓ $P(X) = 0$ EP
✓ $P(X) > 0$ profit
$P(X) < 0$ loss.

الحالة هاي بستثنيها لانو في السؤال معطيني إنو لا يوجد خسارة. noloss.

$X \in [500, 1800]$



∴ → profit $(500, 1800)$

∴ → noloss $[500, 1800]$ ✓

d) What is the price per unite that produces the maximum profit ?

$p = 25 - 0.01X$

$P(X) = -0.01X^2 + 23X - 9000$

$a = -0.01 \quad b = 23 \quad c = 9000$

maximum profit $= P(1150) = 4225 \Rightarrow (1150, 4225)$.

$x = \frac{-b}{2a} = \frac{-23}{-2(0.01)} = 1150$

$p = 25 - 0.01X \rightarrow$ بعوض محل X بالرقم الي طلع او جدته من $x = \frac{-b}{2a}$

$p = 25 - 0.01(1150)$.

$p = 25 - 11.5 = 13.5$ \$.

price $= 13.5$ \$

e). if the price-supply equation for this product is given by :-
$p = 10 + 0.02X$ Find the equilibrum point.

$D: p = 25 - 0.01X$

$S: p = 10 + 0.02X$

EP ⇒ $D = S$

$25 - \cancel{0.01X} = 10 + 0.02X$
$\quad \cancel{+0.01X} \qquad + 0.01X$

$25 = \cancel{10} + 0.03X \rightarrow \frac{15}{0.03} = \frac{\cancel{0.03}X}{\cancel{0.03}}$
$-10 \quad \cancel{-10}$

$X = 500$

$p = 25 - 0.01(500)$.

EP $= (500, 20)$

$p = 25 - 5$

Q2:-

ترتيب المعادلات .

A certain product has supply(1) and demand(2) functions given by $2p - q = 40$ (1) → supply. and $pq = 100 + 2q$ (2), resectively.
→ demand.

a).- if the price is 50$, how many unites are supplied and how many unites are demanded. Is the price likely to increase from $50 or decrease from it Explain.

$S: 2p - q = 40$

$p = 50 \Rightarrow 2(50) - q = 40$

$100 - q = 40$

$-100 \qquad -100$

$-q = -60$

$q = 60$

$D: pq = 100 + 2q$

$p = 50 \Rightarrow 50q = 100 + 2q$

$-2q \qquad -2q$

$\frac{48q}{48} = \frac{100}{48}$

$q = 2.08$

$q_s > q_D \rightarrow$ surplus.

$60 > 2.08$

→ the price will decrease from 50 $.

b).- if a tax of 5$ per item is livied on the supplier, who passes it onto the consumer as a price increase, find the market equilibrium point after the tax.

Tax = 5 $.

$S:- 2p - q = 40$

$\frac{2p}{2} = \frac{40 + q}{2}$

$p = 20 + \frac{1}{2}q$

$S_{new}\ p = 20 + \frac{1}{2}q + 5$ ← Tax.

$S_{new}\ p = 25 + \frac{1}{2}q.$

$EP \Rightarrow S_{new} = D$

$D = pq = 100 + 2q.$

تعويض p

$= (25 + \frac{1}{2}q)q = 100 + 2q.$

$= 25q + \frac{1}{2}q^2 = 100 + 2q$

$-2q \qquad\qquad -2q$

$(23q + \frac{1}{2}q^2 - 100 = 0) * 2$

$q^2 + 46q - 200 = 0$

$(q + 50)(q - 4) = 0$

$q = -50 \rightarrow$ X مرفوضة لانه لايوجد كمية بالسالب

$q = 4 \rightarrow$ ✓

$p = 25 + \frac{1}{2} \cdot 4$

$p = 25 + 2 = 27.$

$EP = (4, 27)$

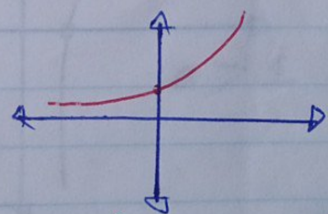
Section 5.1
Exponential and logaritmic function

* For any real number a and b and positive integers m and n.

1. $a^n = a \times a \times a \dots a$ (n times).
2. $a^m a^n = a^{m+n}$
3. For $a \neq 0$, $\frac{a^m}{a^n} = \begin{cases} a^{m-n} & m > n \\ 1 & m = n \\ \frac{1}{a^{n-m}} = & m < n \end{cases}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.
6. $(a^m)^n = a^{m \cdot n}$
7. $a^0 = 1$ $(a \neq 0)$.
8. $a^{-n} = \frac{1}{a^n}$ $(a \neq 0) \rightarrow (a^{-1})^n = a^{-n} = \frac{1}{a^n}$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} \rightarrow (\sqrt[n]{a})^m$ (if n even, $a > 0$)

Exponential functions :- الاقتران الاسي

$y = a^x$ $\quad a \neq 1$ / $a > 0$



* The Domain مجال of the exponential function (the value for which x can be): all real number.
* The reange مدى: all positive number $\rightarrow (0, \infty)$
* For $a > 1$, the function $y = y_0 a^{kx}$ is called the general exponential function.

  $K > 0$ means exponential growth.

  $k < 0$ means exponential decay.

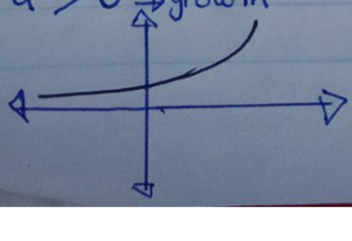
$y = e^x$

$e \approx 2.7$

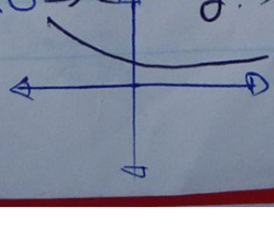
* Special fuction: $f(x) = y_0 e^{kx}$

The shope of the curve will always be:-

$a > 0 \rightarrow$ growth

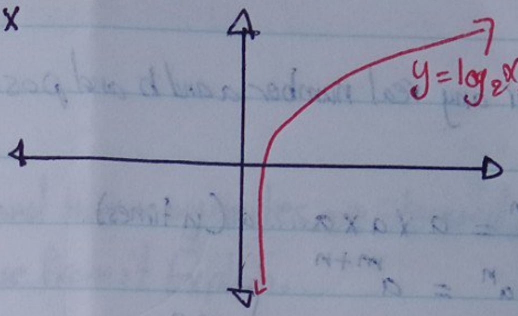


$a < 0 \rightarrow$ (decay.)



* For $a>0$, $x>0$ the function $y=\log_a x$ is called the logerithmic function.

$$y=\log_a x \Rightarrow a^y = x$$



$\log_3 9 = 2$

$3^{\boxed{2}} = 9$

Section 5.2
Logarithmic Function and Their properties.

Def : Logarithmic Function.
For $a>0$ and $a \neq 1$, the logarithmic Function $y = \log_a x$.
has domin $x>0$, bace a and defined by $a^y = x$ (exponantial form).

Example:-

1). $\log_2 8 = 3 \rightarrow 2^3 = 8$

2). $\log_3 27 = 3 \rightarrow 3^3 = 27$

3). $4^3 = 64 \rightarrow$ in logarithmic form $= \log_4 64 = 3$

Doman $\rightarrow x > 0$

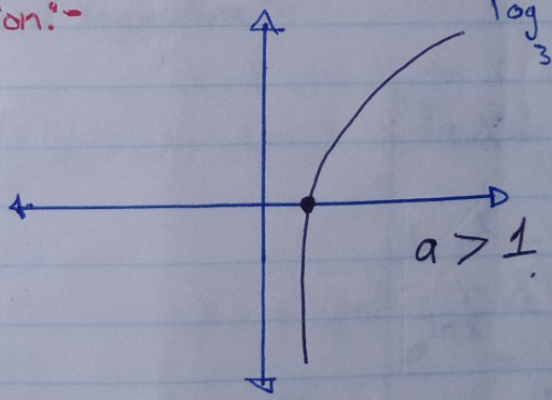
$\log_3 \boxed{} \rightarrow$ يجب أن يكون عدد موجب

* Graphing a logarithmic Function:-

Example :-

$y = \log_2 x$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



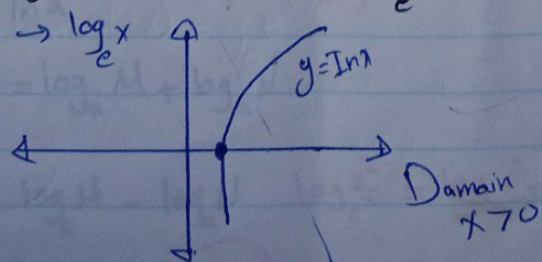
* Common and Natuaral logarithms.
$\rightarrow$ Common logarithms $\log x \rightarrow \log_{10} x$

هذول موجودات على الآلة الحاسبة

$\rightarrow$ Natural logarithms $\ln x \rightarrow \log_e x \rightarrow e \approx 2.7$

$y = \ln x \rightarrow \log_e x$

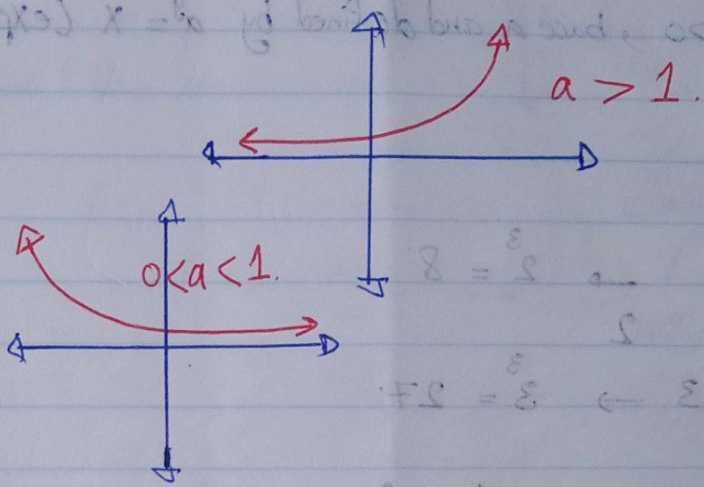
Graph.



* Expontial fuctions :-

$y = a^x$, $a \neq 1$, X real number.

a : base
X : Exponent



a > 1 Growth function

0 < a < 1 decay function

* $a^m \cdot a^n = a^{m+n}$

* $\frac{a^m}{a^n} = a^{m-n}$

* $(ab)^m = a^m \cdot b^m$

* $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

* $(a^m)^n = a^{m \cdot n}$

* $a^{-n} = \frac{1}{a^n}$

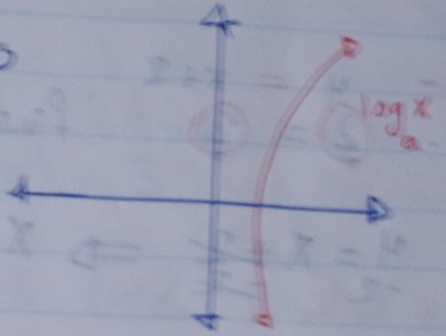
* $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

* Logarithmic Function :-

$y = \log_a x$, $a > 0$, $x > 0$

$y = \log_a x$ if and only if $a^y = x$

Domain: $(0, \infty)$



* Special function

$\log_e x = \ln x$ (if $a = e \approx 2.7$)
↳ (natural logarithm)

$\log_{10} x = \log x$ $\boxed{\log}$ $\boxed{\ln}$

* properties of exponential and logarithms :-

① $\log_a x = y \Leftrightarrow a^y = x$

② $\log_a a^x = x$ $\quad \log_5 5^3 = \boxed{3}$

③ $a^{\log_a x} = x$ $\quad 7^{\log_7 5} = \boxed{5}$

④ $\log_e x = \ln x$ $\quad * \log_a 1 = 0$

⑤ $\log_{10} x = \log x$ $\quad * \log_a a = 1$

⑥ $\log_a b = \frac{\ln b}{\ln a}$ $\quad \log_3 5 = \frac{\ln 5}{\ln 3}$

⑦ $\log_a MN = \log_a M + \log_a N$

⑧ $\log_a \frac{M}{N} = \log_a M - \log_a N$ $\quad \log_5 \frac{x^2}{3} = \log_5 x^2 - \log_5 3$

⑨ $\log_a x^r = r \log_a x$ $\Rightarrow \log_3 5^2 = 2 \log_3 5$

$\ln x = y \Leftrightarrow e^y = x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln ab = \ln a + \ln b$

$\ln \frac{a}{b} = \ln a - \ln b$

$\ln a^r = r \ln a$

$\ln 1 = 0$

$\ln e = 1$

* $a^m = a^n \Rightarrow m = n$

Example :- $4^y = 2^{x+2}$ ... $\textcircled{2} = \textcircled{2}$ find x?

$$\Rightarrow \begin{array}{l} y = x + 2 \\ -2 \qquad \;\; -2 \end{array} \Rightarrow x = 2$$

* $\log_a m = \log_a n$

($\ln m = \ln n$
 $\rightarrow m = n$

$\Rightarrow m = n$

Example :-

$\log_{2}(x+3) = \log_{2}(2x-3)$. Find x?

$\Rightarrow \cancel{x} + 3 = 2x - 3$
$\quad -\cancel{x} \qquad\;\; -x$

$3 = x - 3 \Rightarrow x = 6$
$+3 \qquad +3$

↲ يجب عندما نعوض القيمة x لا يجب ان نعطي قيمة سالبة لانه لا يكون داخل log قيمة سالبة.

Example :- كوزلا يطلب

① $\log x + \log(x+2) = \log(2+x)$.

$\log x(x+2) = \log(2+x)$.

$x^2 + 2x = \cancel{2+x}$
$-2-x \qquad \cancel{-2-x}$

$x^2 + 2x - 2 - x = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ ↲ مرفوضة لانه عندما اعوضها يعطي $\log -2$ و لا يمكن ان يكون log سالب.

$x = 1$ ↳ ناخذها لانها لا تعمل اي مشاكل.

Example :-

(2). $\log_2 (x+10) = 3$.   find X?.

$2^3 = x + 10$

$8 = x + \cancel{10}$
$-10 \quad \cancel{-10}$

$\boxed{x = -2} \rightarrow$

$\log_2(-2+10) = 3$

$\log_2 8 = 3$ (✓)

بعتمدها لأنه عندما عوضنا أعطتني قيمة موجبة داخل log .

(3). $C(x) = 5x^2 + 2x + \log_3 (x^2+9)$. Find the fixed cost ?
$\hookrightarrow C(0) = 0$

$C(0) = 5.0^2 + 2.0 + \log_3 (0^2+9)$

$C(0) = 0 + 0 + \log_3 (9)$.

$\log_3 (9) = \underline{\underline{3^{\boxed{2}} = 9}} \Longrightarrow \frac{\ln 9}{\ln 3} = \boxed{2}$.

* Using Calculator :-

* $2^{1.5} = 2^\wedge 1.5 = \underline{2.83}$ 2.8284 → بقرب لأقرب منزلتين .

* $e^3 = ((\text{shift})(\ln)(3) = 20.09$.

* $\log_{10} 5 = \log 5$ ($\boxed{\log}\ \boxed{5} = 0.699$).

* $\ln 9 = (\ln 9 = 2.197)$.

* $\log_3 7 = \frac{\ln 7}{\ln 3} = (\ln 7 \div \ln 3 = 1.77)$.

Example :-

① - $e^{x} = 5$ Find X :-

بأخذ ln للطرفين حتى تبقى X لوحدها

rule :- $\cancel{\ln e}^{x} = x$.

$= \cancel{\ln e}^{x} = \ln 5$

$x = \ln 5$

$x = 1.61$

② - $2^{x+3} = 5$

$\ln 2^{(x+3)} = \ln 5$

$\ln x^{r} = r \ln x$

$(x+3)\frac{\cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 5}{\ln 2}$

$\underset{-3}{x+\cancel{3}} = \frac{\ln 5}{\ln 2} - 3$

$x = \frac{\ln 5}{\ln 2} - 3$

$x = -0.68$

Section 1.6 and 6.2

Simple and Compound Interest.

[الفائدة عندما تكون بسيطة أو مركبة. (قوانين).

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* Simple Interest :

- if $P in invested at an interest rate of r per years, than the Simple interest, and the future value S after t years, are :-

$$S = P + I$$

$$I = P.r.t$$

→ S :- futur value . المبلغ النهائي.

P :- present value :- . المبلغ الأصلي.

I :- the interest will be earned :- (مبلغ الفائدة) مقدار الفائدة

r :- annual rate :- نسبة الفائدة : $5\% = 0.05$.

t :- time (yeares).

* Example :

if $3000 is invested for 30 months (تحول yeares) at simple interest rate of 5% ?

a. How much interest will be earned ? $I = ??$

$P = 3000$ / $t = 30 \text{ months} = \frac{30}{12} = 2.5 \text{ years}$ $r = 5\% = 0.05$ $I = ??$

$$I = P.r.t$$

$$I = (3000).(0.05).(2.5)$$

$$I = 375\ \$.$$

B). What is the futur value of the investment after $\boxed{30 \text{ months}}$ → 2.5 yeares ? S ?

$P = 3000$ / $I = 375$ / $S = ??$

$$S = P + I$$

$$S = 3000 + 375$$

$$S = 3375\ \$.$$

تكملة السؤال . ⟹

تكملة:

⟹ Example

سؤال عن وقت ← (T)

C) - How long does it take the investment to be worth 7500 $? $t = ??$

$p = 3000 \;/\; r = 0.05 \;/\; t = ?? \;/\; S = 7500$.

$$S = P + I$$

$$S = P + P \cdot r \cdot t$$

$$7500 = \cancel{3000} + 3000 \cdot 0.05 \cdot t$$
$$-3000 \quad \cancel{-3000}$$

$$4500 = (3000) \cdot (0.05) \cdot t$$

$$\frac{4500}{150} = \frac{\cancel{150}}{\cancel{150}} \cdot t$$

$t = 30$ years .

* Compound m times per year :-

$$S = P\left(1 + \frac{r}{m}\right)^{mt}$$

S: futur value.
P: presnt value.
r: annual rate.
t: time (years).
m:- كم مرة في سنة.

* Compounded annually → m = 1 — *كل سنة.
* Compounded semainnually → m = 2 — *كل نص سنة يعني 6 أشهر + 6 أشهر.
* Compounded quarterly → m = 4 — *كل ربع سنة يعني كل 3 أشهر + 3 أشهر + 3 أشهر + 3 أشهر
* Compounded monthly → m = 12. — *كل شهر يعني في سنة 12 شهر.
* Compounded daily → m = 365 — *كل يوم في السنة = 365
* Compounded Continuousely → m = ∞ — *كل لحظة في سنة.

يوجد لها قانون:

$$S = P \cdot e^{rt}$$

* كيفية صياغة الأسئلة عند حل المطلوب:

* What is the future value → S ?
* What anual reat → r ? → 100%
* How much should be deposited / inveted → P ?
* How long → t ?
* How mach interest will be earned (S − P) ?
مقدار الزيادة:

* Example 2

you have $18500 for investment :

محصول

A). what is your ~~futur value~~ if you invest this money for 6 years at on annual reat of 10% compaunded quarterly → S ?

$P = 18500\$$, $t = 6$ yeares $r = 0.1$ $m = 4$

$$S = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$S = 18500\left(1 + \frac{0.1}{4}\right)^{4.6}$$

$$S = 18500(1 + 0.025)^{2.4}$$

$$S = 18500(1.025)^{24}$$

$$S = 33461,43\ \$$$

B). How much interest will be earned ?

$$S - P = 33461,43 - 18500 = 14961,43\ \$.$$

C). How long will it take for your mony to grow to 28000 $ in account paying 7.5% Compounded Continuosy so t = ??

$P = 18500$ / $S = 28000$ / $r = 7.5\% = 0.075$ / $t = ??$

$$S = pe^{rt}$$

$$\frac{28000}{18500} = \frac{\cancel{18500}e^{0.075t}}{\cancel{18500}}$$

$$1.51 = e^{0.075t}$$

$$\ln(1.51) = \cancel{\ln e}^{0.075t}$$

$$\frac{\ln 1.51}{0.075} = \frac{\cancel{0.075}t}{\cancel{0.075}}$$

$t = 5.49$ years

Exampel:-

كان عندي P بستخدمها حتى يصير عندي كم ضعف يعني 2P.

(2) What annual rate of interest you seek if you want to double your investment in 6 years if the amount is compounded monthly? r?

$S = 2P$ / $t = 6$ years / compounded monthly $= 12$.

$$S = P\left(1+\frac{r}{m}\right)^{mt}$$

$$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}}{\cancel{P}}\left(1+\frac{r}{12}\right)^{12 \cdot 6}$$

$$\Rightarrow (2)^{\frac{1}{72}} = \left(\left(1+\frac{r}{12}\right)^{\cancel{72}}\right)^{\cancel{\frac{1}{72}}}$$

$$(2)^{\frac{1}{72}} - 1 = \cancel{1} + \frac{r}{12} \cancel{-1}$$

$$12 \cdot \left(2^{\frac{1}{72}} - 1\right) = \left(\frac{r}{\cancel{12}}\right) \cancel{\cdot 12}$$

$$r = 12 \cdot \left(2^{\frac{1}{72}} - 1\right) \qquad 2^{\frac{1}{72}} = 2^\wedge(1 \div 72)$$

$r = 0{,}1161 \times 100\%$ ← (نسبة الفائدة) لانه r يجب ان تكون بالنسبة المئوية

$r = 11{,}61\ \%$

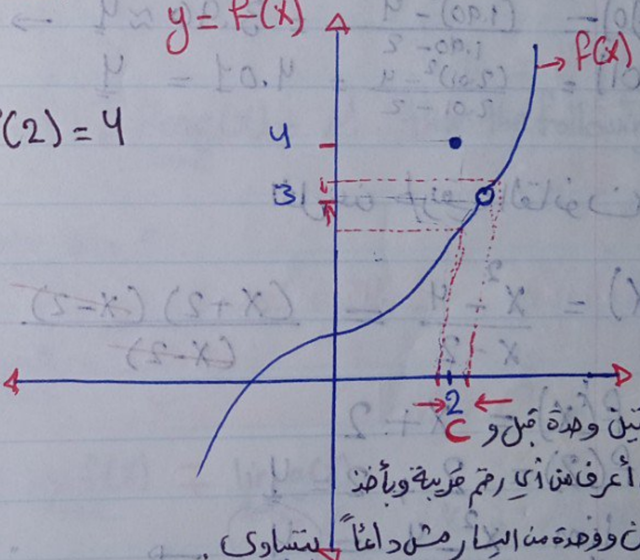
Chapter 9
Derivatives — المشتقات

Section 9.1
Limits (النهايات)

* Let $f(x)$ be a function defined on an open interval containg c, except perhops at c. than $\lim\limits_{x \to c} f(x) = L$

(limit of $f(x)$ as x approches c equals L).
The number L exist if we can make values of $f(x)$ as close to L as we choosing value of x sufficiently close to c.

_ when the value of $f(x)$ don't approch a single finite value L as x approaches c, we say the limit does not exist. (DNE)
أختصارها.



بأخذ نقطتين واحدة قبل و واحدة بعد حتى أعرف من أي رقم قريبة وبأخذ واحدة من اليمين وواحدة من اليسار مش دائماً بتساوى.

* Example 1 :-

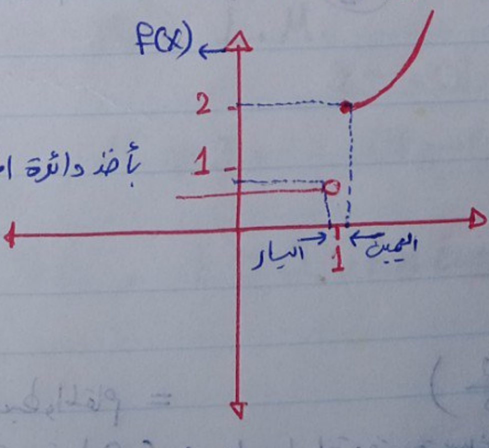
Find $f(2) = 4$ ← صورتها.

$\lim\limits_{x \to 2} f(x) = 3$

$\lim\limits_{x \to 2^+} f(2) = 3$ من جهة اليمين

$\lim\limits_{x \to 2^-} f(2) = 3$ من جهة اليسار

* Example 2 :-



Find :-

① $f(1) = 2$ → صورتها ← بأخذ دائرة المغلقة

② $\lim\limits_{x \to 1} f(x) = ??$

$\lim\limits_{x \to 1^+} f(x) = 2$

$\lim\limits_{x \to 1^-} f(x) = 1$

$\lim\limits_{x \to 1} f(x) = DNE$, $\lim\limits_{x \to 1^+} f(x) = 2 \neq \lim\limits_{x \to 1^-} f(x) = 1$
غير موجودة

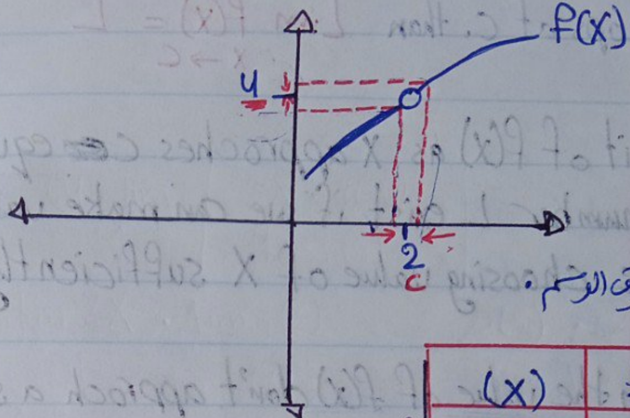
* Example :- 3 .

غير معرفة

Let $f(x) = \frac{x^2-4}{x-2}$, $x \neq 2$ ($f(x)$ is not defined at $x = 2$) . ?

$\lim_{x \to 2} f(x) = 4$

$\lim_{x \to 2^+} f(x) = 4$ $\lim_{x \to 2^-} f(x) = 4$



* الحل عن طريق الرسم

	(x)	f(x)
Left	1.90	3.90
	1.93	3.93
	1.99	3.99
4	2	
	2.01	4.01
right	2.09	4.09
	2.10	4.10

$f(x) = \frac{x^2-4}{x-2}$

$f(1.90) = \frac{(1.90)^2-4}{1.90-2} = 3.90 \approx 4$ → من جهة اليسار

$f(2.01) = \frac{(2.01)^2-4}{2.01-2} = 4.01 = 4$ من جهة اليمين .

4 قريب من 4 .

* الحل عن طريق القانون *

$f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$f(x) = x+2$

$f(2) = 2+2 = 4$

$\Longrightarrow \lim_{x \to 2} \frac{x^2-4}{x-2} = 4$ → عند حل القانون ما بأخذ من اليمين أو من اليسار فقط .

* not $\frac{0}{0}$

$\lim_{x \to 2} \frac{x^2-4}{x-2}$?

$\Rightarrow \frac{2^2-4}{2-2} = \left(\frac{0}{0}\right)$ = نعوض 2 في البسط والمقام

في حال طلع معي عند التعويض $\left(\frac{0}{0}\right)$ بحل معادلة بفتح قوسين وبأختصر حتى أقدر أحل.

* One side limit :-

- limit from right :-

$\lim_{x \to c^+} f(x) = L$, f(x) approch the value L as x approch c from the right .

- limit from left :-

$\lim_{x \to c^-} f(x) = L$, $f(x) \to L$ as $x \to c^+$

if $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) \Rightarrow \lim_{x \to c} f(x) = L \Rightarrow$ exists موجود

if $\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x) \Rightarrow$ Does not exists (DNE) . غير موجود

* Properties of limits :-

if k constant (رقم) , $\lim_{x \to c} f(x) = L$, $\lim_{x \to c} g(x) = M$ then the following are rule. قوانين

1. $\lim_{x \to c} k = k$ → رقم ثابت Ex:- $\lim_{x \to 2} 5 = 5$

2. $\lim_{x \to c} x = c$ Ex:- $\lim_{x \to 4} x = 4$

3. $\lim_{x \to c} [f(x) \mp g(x)] = (L \mp M) \Rightarrow \lim_{x \to c} f(x) \mp \lim_{x \to c} g(x)$

4. $\lim_{x \to c} f(x) \cdot g(x) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = L \cdot M.$

5. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{M}$, $M \neq 0$

6. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} = \sqrt[n]{L}$, $L > 0$ if n even. عدد زوجي

Ex:- $\lim_{x \to 2} \sqrt{x^2+1} = \sqrt{2^2+1} = \sqrt{5}$

Ex:- $\lim_{x \to 1} \frac{x}{x-1} = \frac{1}{1-1} = \frac{1}{0} \Rightarrow$ DNE → Becous limit → $(-\infty, \infty)$ ← يعني مستحيل

$\lim_{x \to 1^-} \frac{x}{x-1} \neq \lim_{x \to 1^+} \frac{x}{x-1}$ يتساوي limit لذلك غير موجودة النهاية.

Example :- Q52 page 554.

if $\lim\limits_{x\to5}[f(x) - g(x)] = 8$ and $\lim\limits_{x\to5} g(x) = 2$

Find:-

1- $\lim\limits_{x\to5} f(x)$

$= \lim\limits_{x\to5} f(x) - \lim\limits_{x\to5} g(x) = 8$

$\to \lim\limits_{x\to5} f(x) - \cancel{2} = 8$
$\quad\quad\quad\quad\quad \cancel{+2} \quad\; +2$

$\lim\limits_{x\to5} f(x) = 10$

2- $\lim\limits_{x\to5} 3f(x)$?

$3 . \lim\limits_{x\to5} f(x) = 3.10 = 30$

3- $\lim\limits_{x\to5} [g(x)^2 - f(x)]$

$= \left(\lim\limits_{x\to5} g(x)\right)^2 - f(x)$

$= 2^2 - 10 = 4 - 10 = -6$

$\left(\lim\limits_{x\to5} g(x)\right)^2 = \lim\limits_{x\to5} g(x) \cdot \lim\limits_{x\to5} g(x) = 2.2 = 4$

4. $\lim\limits_{x\to5} \left[\dfrac{2x \cdot g(x)}{4 - f(x)}\right]$

$= \dfrac{\lim\limits_{x\to5} 2x \cdot \lim\limits_{x\to5} g(x)}{\lim\limits_{x\to5} 4 - \lim\limits_{x\to5} f(x)} = \dfrac{2.(5) \cdot 2}{4 - 10}$

$= \dfrac{20}{-6} = -\dfrac{10}{3}$

* $\lim_{x \to c} f(x) = f(c)$

if $f(x)$ is a polynomial function → كثير حدود.

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

هو أي عدد عليه قوة مثل $x^3 + 2x + 1$ أو $x^0 = 1$ وثابت بقدر أعوض مباشرة فيه.

Example :-

$\lim_{x \to -1} x^3 + 2x + 3 = (-1)^3 + 2(-1) + 3 =$
$-1 - 2 + 3 = 0$

* $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}, \quad g(c) \neq 0$

$f(x)$ and $g(x)$ are polynomials

$\frac{f(x)}{g(x)}$ rational function

* عندما أعوض القيمة بتظهر عندي أحد الحالات الآتية :-

$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$
- → $\left(\frac{0}{a}\right) = 0$
- → $\left(\frac{a}{b}\right) = \frac{a}{b}$
- → $\left(\frac{a}{0}\right) = \text{DNE} \to (\infty, -\infty)$
- → $\left(\frac{0}{0}\right)$ → factoring في هذه الحالة نحل البسط والمقام

Example :-

1. $\lim_{x \to 2} x^2 - 1 = 2^2 - 1 = 4 - 1 = 3$

2. $\lim_{x \to +1} \frac{x^2 + 3x}{x - 2} = \frac{1^2 + 3(1)}{1 - 2} = \frac{4}{-1} = -4.$

3. $\lim_{x \to 4} \frac{x - 4}{x + 3} = \frac{4 - 4}{4 + 3} = \frac{0}{7} = 0$

* Example :

4) $\lim\limits_{x \to 4} \frac{x^2+3}{x-4} = \frac{4^2+3}{4-4} = \frac{16+3}{0} = \frac{19}{0}$ (DNE) هـ عندما لا يحدد لي من جهة اليسار أو اليمين
حدودي من جهة اليمين أو اليسار يختلف الجواب مثلاً :-

$\lim\limits_{x \to 4^-} \frac{x^2+3}{x-4} = \frac{(3.9)^2+3}{3.9-4} = \frac{18.21}{-0.1} = -\infty$ طبعاً عندما يكون من جهة اليسار بأخذ رقم قريب على 4 مثلاً 3.9/3.8

من جهة اليسار

$\lim\limits_{x \to 4^+} \frac{x^2+3}{x-4} = \frac{(4.1)^2+3}{4.1-4} = \frac{19.81}{0.1} = +\infty$ طبعاً عندما يكون من جهة اليمين بأخذ رقم يكون قريب لـ 4 (4.1/4.2/4.3)

من جهة اليمين

* عندما تكون الاشارة سواء كانت (سالب أو موجب)، إذا كانت قبل الرقم فتدل على أن رقم سالب أو موجب العدد، أما إذا كانت بعد الرقم تدل على أن الجهة التي يقترب منها العدد سواء كانت (اليمين أو اليسار).

5. $\lim\limits_{x \to 2} \frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{4-4}{2-2} = \frac{0}{0}$ Indeterminal form (تحليل الـ ~~قوس~~ ..)

$= \lim\limits_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim\limits_{x \to 2} (x+2) = 2+2 = 4.$

6) $\lim\limits_{x \to 4^-} \frac{3x^2}{4-x} = \frac{3 \cdot (3.9)^2}{4-3.9} = \frac{45.63}{0.1} = +\infty$

7) $\lim\limits_{x \to -2} \frac{x^2+2x}{x+2} = \frac{(-2)^2+2(-2)}{-2+2} = \frac{4-4}{-2+2} = \frac{0}{0} \to$ نقوم بإخراج عامل مشترك.

$\lim\limits_{x \to -2} \frac{x\cancel{(x+2)}}{\cancel{x+2}} = \lim\limits_{x \to -2} x = -2$ (common factroe).

* Peicewise function :- إقتران متعدد القاعدة

$$f(x) = \begin{cases} x^2+1, & x \leq 1 \\ x+2, & x > 1 \end{cases}$$

Find :- $2 > 1$

① $\lim_{x \to 2} f(x) = \lim_{x \to 2} x+2 = 2+2 = 4.$

② $\lim_{x \to 0} f(x) \overset{0 \leq 1}{=} \lim_{x \to 0} x^2+1 = 0^2+1 = 1$

③ $\lim_{x \to 1} f(x) = ??$ → طبعاً رح أعوض في الجهتين يعني يمين ويسار بسبب ان العدد واحد اتى كطرف قاعدي

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x+2 = 1+2 = 3.$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2+1 = (1)^2+1 = 2$

$\neq$

$\lim_{x \to 1} f(x) = DNE$ ← يعني (lim) عند 1 غير موجود بس عند باقي الارقام موجود

$f(1) = 1^2 + 1 = \underline{\underline{2}} \rightarrow f(1) = 2$ نعوض في المساواة فقط عندما يطلب من الصورة $f(x)$

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Section(9.2)

Continuous function :- الأقترانات المتصلة

* The function $f(x)$ is continuos at $x = c$ if all the following Conditions are satisfied :-

* الشروط التي تحدد اذا كان الأقتران متصل أو لا .

1. $f(c)$ exists :- يجب أن يكون الاقتران معرف عند c .

2. $\lim_{x \to c} f(x)$ exists :- * يجب أن يكون (lim) موجود عند تقترب x من c

3. $\lim_{x \to c} f(x) = f(c)$. *ويجب أن تتساوى الصورة مع lim

* اذا تحققت الشروط الثلاث يعني أن الاقتران متصل

* اما اذا خلت واحدة من الشروط الثلاث يصبح الاقترن غير متصل .

* if one or more of continuous abou do not hold, we say the function is discontinuous at $x = c$.

Example :- حددي المثال اذا كان متصل أو غير متصل .

$$f(x) = \begin{cases} x^2+1, & x \leq 1 \\ x+2, & x > 1 \end{cases}$$

Find :-

1). $f(1) = x^2 + 1 = 1^2 + 1 = 2 \Rightarrow f(1) = 2$

2) $\lim_{x \to 1} f(x) =$

$\lim_{x \to 1^-} f(x) = x^2 + 1 = 1^2 + 1 = 2$

$\lim_{x \to 1^+} f(x) = x + 2 = 1 + 2 = 3$

$\lim_{x \to 2} f(x) = DNE$.

$f(1) \neq \lim_{x \to 1} f(x)$ $\therefore$ dis Continuous

الأقتران غير متصل .

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* Not :-

1). if $f(x)$ is polgonmial, than $f(x)$ is Continuous for all x $\left(\lim_{x \to c} = f(c)\right)$

* دائماً اقتران متعدد القاعدة متصل. مثل :-

$$f(x) = x^3 - x$$

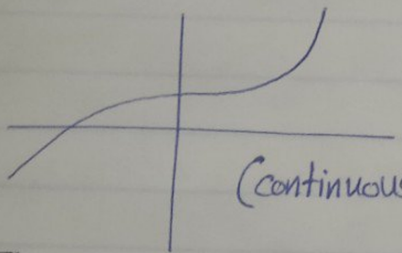
$f(1) = 1^3 - 1 = 0$

$\lim_{x \to 1} f(x) = x^3 - 1 = 1^3 - 1 = 0$ $\quad f(1) = \lim_{x \to 1} f(x)$ Continuous.

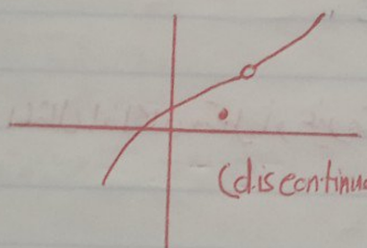
2). $\frac{f(x)}{g(x)}$ (rational function) is Continuous for all X such that $g(x) \neq 0$

الكسور جميعها متصل ما عدا الارقام التي تجعل المقام يصبح صفر مثل :-

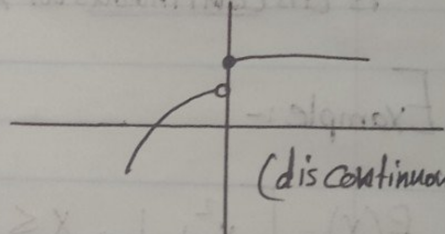
$y = \frac{x^2 - 2}{x + 3} \rightarrow$ Continuous $(-\infty, \infty) - \{-3\}$.



(continuous)



(discontinuous)



(discontinuous.)

Example: for waht values of x, are the follwing funchion are continuous.

1) $h(x) = \frac{3x + 2}{4x - 6} \Rightarrow$ Continuosc for all x except $x = \frac{3}{2}$ عامل

$(4x - 6 = 0 \Rightarrow \frac{4x}{4} = \frac{6}{4} \Rightarrow x = \frac{3}{2}) \Rightarrow (-\infty, \infty) - \{\frac{3}{2}\}$.

2). $f(x) = x^3 - 2 \Rightarrow$ Continuous for all x, $(-\infty, \infty)$.

3). $g(x) = \frac{x^2 - x - 2}{x^2 - 4} \Rightarrow$ Continuous for all x except $x^2 - 4 = 0$

$(-\infty, \infty) - \{-2, 2\}$.

$\hookrightarrow (x - 2)(x + 2) = 0$

$2 = x \; / \; -2 = x$

$\hookrightarrow$ (discontinuous).

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Example 2:-

Determine the values of x, if any, for which the following functions are discontinuous.

① $f(x) = \begin{cases} 4-x^2, & x < 2 \\ x-2, & x \geq 2 \end{cases}$ → $x = 3$ → Continuous.

متصل لانه كثير حدود

متصل.

- if $x<2$, $f(x) = 4-x^2$ is continuous for all x.
- if $x>2$, $f(x) = x-2$ is continuous for all x.
- if $x=2$ (3 condition) → يوجد ثلاث شروط.

1- $f(2) = 2-2=0$

2). $\lim_{x\to 2} f(x) = 0$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x-2 = 2-2=0$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4-x^2 = 4-2^2=4-4=0$

3). $\lim_{x\to 2} f(x) = f(2)$.

$0 = 0$ $f(x)$ is continuous for all x $(-\infty,\infty)$.

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2). $f(x) = \begin{cases} x^2+1, & x \boxed{>-1} \\ 2x+1, & x \boxed{\leq -1} \end{cases}$ → Contnuous $x > -1$ → contnuous $x < -1$

متصل لانه كثير حدود.
متصل لانه كثير حدود.

at $= -1$

1). $f(-1) = 2(-1)+1 = -2+1 = -1$

2). $\lim_{x \to -1} f(x) = DNE$

$$\left.\begin{aligned} \lim_{x \to -1^+} f(x) &= \lim_{x \to -1^+} x^2+1 = (-1)^2+1 = 2 \\ \lim_{x \to -1^-} f(x) &= \lim_{x \to -1^-} 2x+1 = 2 \cdot -1 + 1 = -2+1 = -1 \end{aligned}\right] \neq$$

$f(x)$ is not continuous at $x = -1$
$f(x)$ is continhuous $(-\infty, \infty) - \{-1\}$.

$\lim\limits_{x \to \infty, -\infty} \frac{f(x)}{g(x)}$:-

* يوجد ثلاث حالات على هذه القاعدة .

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dagree:- هو أعلى قوة لـ $\underline{x}$

what is the dagree:- 1). $x^2 + 2x + x^3 = 3$ / $2x - x^2 = 2$

1.) degree $f(x)$ > degree $g(x)$ = $(-\infty, \infty) \Rightarrow$ DNE

2.) degree $f(x)$ < degree $g(x)$ = 0

3.) degree $f(x)$ = degree $g(x)$ $\Rightarrow$ dagree "بأخذ معامل x في f(x) على معامل x في g(x) ضمنها

Example:-

1) $\lim\limits_{x \to \infty} \frac{2x^2 + 3x}{x - 1}$ = degree $2 > 1 \Rightarrow$ DNE $(-\infty, \infty)$

2) $\lim\limits_{x \to -\infty} \frac{x^2 + 3x}{1 - x^3}$ = degree $2 < 3 = 0$

3) $\lim\limits_{x \to \infty} \frac{x^2 - 3x + x^3}{2x - 3x^3 + 1}$ = degree $3 = 3 \to \frac{1}{-3}$ → . بأخذ عامل اعلى dagree في البسط على المقام

4) $\lim\limits_{x \to -\infty} \frac{x^5 + 4x^3 - 4}{x^3 + 4x - 2x^5}$ = degree $5 = 5 = \frac{1}{-2} = \frac{-1}{2}$

not:—

degree → للعدد الثابت هو صفر

$\frac{3}{5} = \frac{3^0}{5^0}$

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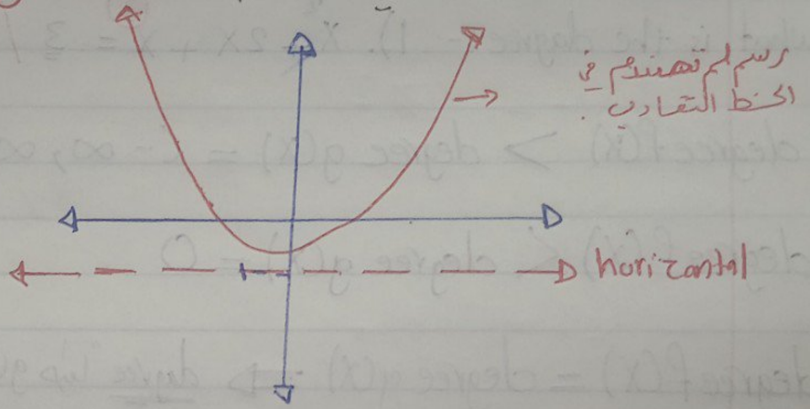
* Horizantal asymptote:- خط التقارب

هو خط متقطع في الرسم لم يكون متصل ويكون خط أفقي (هو خط وهمي).

معادلة الخط الافقي دائماً $\Leftarrow$ $y =$ the number

1.) $\lim\limits_{x \to \infty} \frac{x^2 + 2x}{x - x^2} = -1$

(horizantal asymptote) $= y = -1$



2) $\lim\limits_{x \to -\infty} \frac{x^3 + 2}{x + x^2} = DNE$

horizantal asymptote $\rightarrow$ لا يوجد لدنا جواب (DNE)

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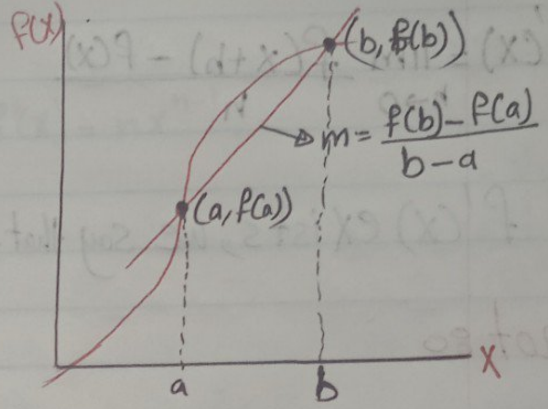
section 9.3
Rates of change and Derivatives

متوسط التغير

* average rate of change of $f(x)$ from $x=a$ to $x=b$ is defined by

$$\text{average rate of change} = \frac{f(b)-f(a)}{b-a}$$

(slope of the segment (or secont line))



Example:- Total cost

$C(x) = 0.01x^2 + 25x + 1500$

Find the avarge rate of cahange of total cost for the second 100 unite produced (from 100 to 200)

a

b

$$\text{average reat of } C(x) = \frac{C(200) - C(100)}{200 - 100} =$$

$$= \frac{(0.01(200)^2 + 25(200) + 1500) - (0.01(100)^2 + 25(100) + 1500)}{200 - 100}$$

$$= \frac{6900 - 4100}{100} = 28 \text{ dollars per unite.}$$

* average rate of change $\longrightarrow$ slop secant line $\rightarrow$ (هو عبارة عن ميل القاطع (متوسط تغير))

* instantanuous reat of change (rate of change) = derivative (slope of the tangent)

تغير اللحظي

ميل المماس (هو نفس المشتقة)

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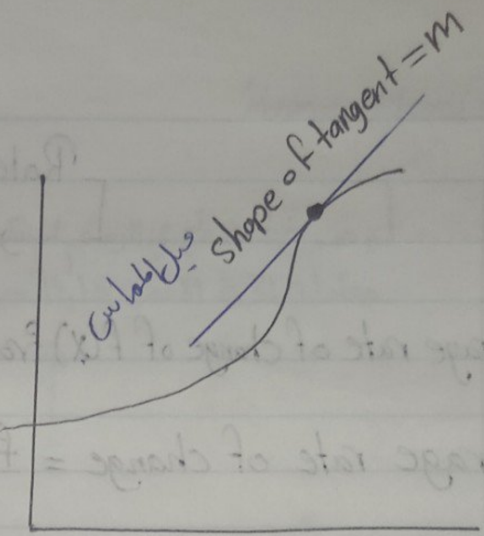
* Derivalive :- الاشتقاق



if f a function defined by $y = f(x)$,
than the derivative of f(x) at any value x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if $f'(x)$ exists, we say that f is differntioble.

* not :

Each of the following menas "find the derivative".

1) Find the instontaneous rate of change . $f'(x)$

2) Find rate of change : . ميل المماس

3). Find the marginal revenue/cost/profit.

$C(x) = 4x + 1$

$MC = 4$

4) Find the slope of the langent line.

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Section 9.4
Derivative Formulas.

Given $f(x)$, y
Find $f'(x)$, $\frac{dy}{dx} = y'$ مشتقة

*** Rule 1:-**
if $f(x) = x^n$, n any real number then $f'(x) = nx^{n-1}$.

Example:-

1) find $f'(x)$ if $f(x) = x^{10}$
$\rightarrow f'(x) = 10x^9$

2) $f(x) = x^{-2}$ Find $f'(x) =$
$f'(x) = -2x^{-3} \Rightarrow \frac{-2}{x^3}$

3) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$
$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

4) $f(x) = \frac{1}{x^3} = x^{-3}$
$f'(x) = -3x^{-4} \Rightarrow \frac{-3}{x^4} =$

not:-
$\sqrt[n]{x} = x^{\frac{1}{n}}$
$x^{-n} = \frac{1}{x^n}$

*** Rule 2:-**
if $f(x) = c$ (constant) than $f'(x) = 0$ مشتقة أي عدد ثابت تساوي صفر

Example:- find the derivative :-

1- $f(x) = 1.5$
$f'(x) = 0$

2- $f(x) = -10.35$
$f'(x) = 0$

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* Rule 3:-

if $f(x) = c.u(x)$, c (constant) / $u(x)$ is differentiable of $x \to$ then $f'(x) = c.u'(x)$.

مابتأثر في الاشتقاق

$$y = 3x^2 \Rightarrow y' = 3(2x) = 6x.$$

* $f(x) = cx \Rightarrow f'(x) = c$

Example:-

1- $f(x) = 2x$

$f'(x) = 2.(1x)^{1-1} = 2x^0 = 2$

2. $f(x) = \frac{3}{\sqrt{x}} = 3x^{-\frac{1}{2}}$

$f'(x) = 3.\frac{-1}{2}x^{-\frac{1}{2}-1} = \frac{-3}{2}x^{\frac{-3}{2}} = \frac{-3}{2\ x^{\frac{3}{2}}} \Rightarrow \frac{-3}{2\sqrt[2]{x^3}}$

Rule $\Rightarrow a^{\frac{m}{n}} = \sqrt[n]{a^m}$

* Rule 4:-

if $f(x) = u(x) \mp v(x)$

than $\Rightarrow f'(x) = u'(x) \mp v'(x)$

Example:-

1) $y = 5x + 3$

$y' = 5 + 0 = 5$

2) $y = 2x^3 + 3x - 5$

$y' = 2(3x^2) + 3 - 0$

$y' = 6x^2 + 3$

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Example:-

1) Find the derivative of $y = 5x^{-3}$

$y' = 5(-3x^{-4}) = -15x^{-4} = \frac{-15}{x^4}$

2) Find the manginal revenue of $R(x) = 15x + 20$

$MR = R'(x) = 15$ (the revenue of producing and selling one additional unite is about $15 approximatly).

تقريباً

مطلوب مشتقة

3) Suppose the total cost is given by $C(x) = 4000 + 55x + 0.1x^2$

- Find the reate of change of $C(x) =$ at $x = 10$.

مشتقة

$C(x) = 4000 + 55x + 0.1x^2.$

$C'(x) = 0 + 55 + 0.1(2x).$

$C'(x) = 55 + 0.2x$

Find the margenal cost of $x = 10$.

$MC = C'(10) = 55 + 0.2(10).$

$MC = C'(10) = 55 + 2 = 57$

$MC = C'(10) = 57$. (the cost of prodecing one additionlunit is about 57).

معادلة المماس

4) Write the quation of tangent line of $y = x^4 - 4x^3 - 2$ at $x = 2$

quation of tangent line $\Rightarrow y - y_1 = m(x - x_1)$. $\boxed{m = \text{slope}}$ / (x_1, y_1) point on the curve.

المشتقة.

at $x = 2 \Rightarrow y = 2^4 - 4 \cdot 2^3 - 2$

$y = 16 - 32 - 2 = -18 \Rightarrow (2, -18)$ (x_1, y_1)

$y = x^4 - 4x^3 - 2.$

$y' = 4x^3 - 12x^2 - 0$

$y'(2) = 4 \cdot 2^3 - 12 \cdot 2^2 =$

$y'(2) = 32 - 48 = -16$

$m \rightarrow$ slope $- y'(2) = -16.$

Equation معادلة $(2, -18)$ $m = -16.$

$= y - y_1 = m(x - x_1).$

$y - -18 = -16(x - 2).$

$y + 18 = -16x + 32$

$ -18 \qquad\qquad -18$

$y = -16x + 14$

(x, y) إحداثيات الأزواج المرتبة

مماس أفقي .

5) Find the coordinates of point where the graph of f(x) has a horizontal tangent ؟

$(x, y) \Rightarrow f(x) = 3x^5 - 5x^3 + 2$

* الخط الأفقي ميله بيكون يساوي (صفر)

مماس أفقي .

هو مشتقة .

* horizontal tangent → slope = 0 / $f'(x) = 0$

$$* \; f(x) = 3x^5 - 5x^3 + 2$$
$$f'(x) = 3(5x^4) - 5(3x^2) + 0$$
$$f'(x) = 15x^4 - 15x^2$$
$$0 = 15x^4 - 15x^2$$
$$0 = 15x^2(x^2 - 1)$$
$$\Rightarrow \frac{15x^2}{15} = \frac{0}{15} \qquad x^2 - 1 = 0$$
$$x = 0 \qquad x = 1 \text{ or } x = -1$$

$x = 0, 1, -1 \Rightarrow y$ بعوض في المعادلة الأصلية حتى أحصل على y

$\rightarrow f(x) = 3x^5 - 5x^3 + 2$

$f(0) = 3.0^5 - 5.0^3 + 2 = 2 \Rightarrow (0, 2)$

$f(1) = 3.1^5 - 5.1^3 + 2 = 0 \Rightarrow (1, 0)$

$f(-1) = 3.-1^5 - 5.-1^3 + 2 = 4 \Rightarrow (-1, 4)$

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6) Suppose that the demand for aproduct given by :-

$$D(P) = \frac{500000}{P^2} - \frac{1}{2} \quad , P > 0$$

* Find and explain the meaning of rate of change (مشتقة) of demand with respect to price when $p = 50$

reat of change $= D'(P) = \frac{dq}{dp} \Rightarrow$ يعني مشتقة

$$D(P) = \frac{500000}{P^2} - \frac{1}{2}$$

إعادة ترتيب للمعادلة :-

$$D(P) = 500000P^{-2} - \frac{1}{2}$$

$$D'(P) = 500000(-2)P^{-3} - 0$$

$$D'(P) = -1000000P^{-3} = \frac{-1000000}{P^3}$$

$$D'(50) = \frac{-1000000}{(50)^3} = -8$$

when price (السعر) increusel (تزيد) by \$1 the quintity damand decreased (تقل) by approximatly (تقريبا) 8 units

* عندما استخدم المشتقة لتفسير يجب ان أكتب (approximatly or about) لانو يأتي بشكل تقريبي وليس صحيح 100%.

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* Averge cost function :- معدل التكاليف .

* if $C(x)$ the total cost than the average cost $\Rightarrow \overline{C(x)} = \frac{C(x)}{x}$ قانون ثابت لمعدل التكاليف
(averge cost.) كرمز

Example :- Given.

$C(x) = 40500 + 190x + 0.2x^2$ Find :-

1- the averge cost functoian :-

$\overline{C}(x) = \frac{C(x)}{x} = \frac{40500 + 190x + 0.02x^2}{x}$

$= \frac{40500}{\cancel{x}} + \frac{190\cancel{x}}{\cancel{x}} + \frac{0.02x^{\cancel{2}}}{\cancel{x}}$

not : $\frac{a+b}{x} = \frac{a}{x} + \frac{b}{x}$

$\overline{C}(x) = 40500x^{-1} + 190 + 0.02x$

2. Find the instantenuose rate of ~~ch~~ change of $\overline{C}(x) \Rightarrow (\overline{C}(x))'$? مشتقة .

$\overline{C}(x) = 40500x^{-1} + 190 + 0.02x$

$(\overline{C}(x))' = 40500(-1)x^{-2} + 0 + 0.02$

$(\overline{C}(x))' = \frac{-40500}{x^2} + 0.02$

Yumna Marei 1200563 Section 9.5

The product Rule and Quotient Rule :- (مشتقة الضرب والقسمة)

* Product Rule :- مشتقة الضرب

if $f(x) = u(x) \cdot v(x)$ where u and v are differentiable function of x, than.

$$f'(x) = u(x) \cdot v'(x) + v(x) \cdot u'(x)$$

* مشتقة الضرب = الأول . مشتقة الثاني + الثاني . مشتقة الأول.

Example :-

1) Find the $\frac{dy}{dx}$ if $y = (2x^3 + 3x + 1)(x^2 + 4)$

$$\frac{dy}{dx} = y' = (2x^3 + 3x + 1) \cdot (2x) + (x^2 + 4) \cdot (6x^2 + 3)$$
$$= 4x^4 + 6x^2 + 2x + 6x^4 + 3x^2 + 24x^2 + 12$$
$$= 10x^4 + 33x^2 + 2x + 12.$$

2). Find the slope of tangent to the graph :

$y = (4x^3 + 5x^2 - 6(x) + 5)(x^3 - 4x^2 + 1)$ at $x = 1$

Slope at $x = 1 \Rightarrow y'(1)$.

$$= y' = (4x^3 + 5x^2 - 6x + 5) \cdot (3x^2 - 8x) + (x^3 - 4x^2 + 1) \cdot (12x^2 + 10x - 6)$$
$$y'(1) = (4(1)^3 + 5(1)^2 - 6(1) + 5) \cdot (3(1)^2 - 8(1)) + ((1)^3 - 4(1)^2 + 1) \cdot (12(1)^2 + 10(1) - 6)$$
$$y'(1) = (4 + 5 - 6 + 5) \cdot (3 - 8) + (1 - 4 + 1) \cdot (12 + 10 - 6).$$
$$y'(1) = 8 \cdot (-5) + (-2 \cdot 16)$$
$$y'(1) = -40 + -32 = -72.$$

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* Quotient Rule :- مشتقة القسمة

if $f(x) = \frac{U(x)}{V(x)}$, $V(x) \neq 0$ than :-

$$f'(x) = \frac{V(x) \cdot U'(x) - U(x) \cdot V'(x)}{(V(x))^2}$$

* مشتقة القسمة = $\frac{\text{المقام . مشتقة البسط} - \text{البسط . مشتقة المقام}}{(\text{المقام})^2}$.

Example :-

1). if $f(x) = \frac{x^2 - 4x}{x+5}$, find $f'(x)$.

$$f'(x) = \frac{(x+5) \cdot (2x-4) - (x^2 - 4x) \cdot (1)}{(x+5)^2}$$

$$= \frac{2x^2 - 4x + 10x - 20 - x^2 + 4x}{(x+5)^2} = \frac{x^2 + 6x + 4x - 20}{(x+5)^2} = \frac{x^2 + 10x - 20}{(x+5)^2}.$$

2). if $f(x) = \frac{x^3 - 3x^2 + 2}{x^2 - 4}$ find the instantanceus rate of change of $f(x)$ at $x = 3$

$\rightarrow f'(3) = ??$

$$= f'(x) = \frac{(x^2 - 4) \cdot (3x^2 - 6x) - (x^3 - 3x^2 + 2) \cdot (2x)}{(x^2 - 4)^2}$$

$$= f'(3) = \frac{(3^2 - 4) \cdot (3 \cdot (3)^2 - 6(3)) - (3^3 - 3 \cdot 3^2 + 2) \cdot (2 \cdot 3)}{(3^2 - 4)^2}$$

$$= \frac{45 - 12}{25} = \frac{33}{25} \approx 1.32.$$

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Example:-

$$R(x) = \frac{60x^2 + 74}{2x+2}$$

a) Find the marginal revenue when 49 unite are sold Interpert your answer.
(Find the revenue from the sold of the 50 th unite). ? $R'(49)$.

$$MR = R'(x) = \frac{(2x+2)\cdot(120x) - (60x^2+74)\cdot(2)}{(2x+2)^2}$$

$$R'(49) = \frac{(2(49)+2)\cdot(120\cdot 49) - (60\cdot(49)^2+74)\cdot 2}{(2\cdot 49+2)^2}$$

$$R'(49) = \frac{(100)\cdot(5880) - (144134)\cdot(2)}{(100)^2}$$

$$R'(49) = \frac{588000 - 288268}{10000} = 29.97 \$.$$

* The revenue of producing and selling one additional unite is about 29.97.

b). Find $R(50) - R(49) = ??$

$$R(50) = \frac{60(50)^2+72}{2\cdot 50+2} = 1471.31$$

$$R(49) = \frac{60(49)^2+72}{2\cdot 49+2} = 1441.34$$

$R'(49) \approx 1471.31 - 1441.34 = 29.97 \$$.
the revenue of producing and selling one additonal unite is 29.97 (the exact revenue).

not:-

$MR = R'(49) \approx R(50) - R(49)$

$R'(45) \approx R(46) - R(45)$

Suction 9.6 Yumna Marei 1200563

قاعدة السلسلة (تركيب الاقترانات) The chain Rule and the prower Rule

* Composite function :-

$\rightarrow (f\circ g)(x) = f(g(x))$

$\rightarrow (g\circ f)(x) = g(f(x))$

Example :-

* $f(x) = 3x^2$, $g(x) = 2x-1$ Find $f(g(x))$:-

* $f(g(x)) = 3(2x-1)^2 = 3(4x^2-4x+1) = 12x^2 - 12x + 3$.

* Find $(f(g(x)))' = (3(2x-1)^2)' = \underline{24}x - 12 \Rightarrow 6(2x-1)\cdot 2 = 12(2x-1) = 24x-12$

if $y = f(g(x))$ than $y' = f'(g(x))\cdot g'(x)$. $\Rightarrow$ قاعدة (مهمة)

Example :-

1) let $f(2) = 1$, $g(1) = 3$, $g'(1) = 5$, $f'(3) = 2$ and $h(x) = f(g(x))$ Find $h'(1)$?

$h(x) = f(g(x))$

$h'(x) = f'(g(x))\cdot g'(x)$

$h'(1) = f'(g(1))\cdot g'(1)$.

$h'(1) = f'(g(1))\cdot g'(1)$.

$h'(1) = f'(3)\cdot 5$

$h'(1) = 2\cdot 5 = 10$

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Example :-

1) if $y = (2x^2 + 3)^2$ find $\frac{dy}{dx}$ $\Rightarrow$ $f(x) = x^2$ / $g(x) = (2x^2 + 3) = f(g(x))$

$\frac{dy}{dx} = y' = \underbrace{2(2x^2+3)}_{f'(g(x))} \cdot \underbrace{(4x)}_{g'(x)}$ مشتقة داخل القوس

$f'(x) = 2x$

$f'(g(x)) = f'(2x^2+3) = 2(2x^2+3)$

2) if $y = \sqrt{x^2-1}$ Find y' :-

$y = (x^2-1)^{\frac{1}{2}}$

$y' = \frac{\cancel{1}}{\cancel{2}} (x^2-1)^{-\frac{1}{2}} \cdot (\cancel{2}x)$ مشتقة داخل القوس

$y' = \frac{x}{(x^2-1)^{\frac{1}{2}}} = \frac{x}{\sqrt{x^2-1}}$

* A special case of chain Rule is the power Rule if $y = u^n$, u function of x

$y' = un^{n-1} \cdot \frac{du}{dx}$.

* Example :- Find the derivative of y.

1) $y = x^{-3}$

$y' = -3x^{-4} = \frac{-3}{x^4}$

Example :- Yumna Mare'i 2005

2)- $y = \frac{x^2+1}{x}$

$y = \frac{x^2}{x} + \frac{1}{x} = x^2 \cdot x^{-1} + x^{-1} = x + x^{-1}$

$y = x + x^{-1}$

$y' = 1 + -1\,x^{-2} = 1 + \frac{-1}{x^2}$

or :- طريقة ثانية عن طريق القسمة والمباشرة. -:

$y' = \frac{x\cdot(2x) - (x^2+1)\cdot(1)}{(x)^2} = \frac{2x^2 - x^2 - 1}{\cancel{x^2}} = \frac{x^2-1}{x^2} = 1 - \frac{1}{x^2}$

3)- $y = x^{\frac{1}{2}}(x^2+1)$ product Rule

$= x^{\frac{1}{2}}\cdot(2x) + (x^2+1)\,\frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}=\frac{-1}{2}}$

$= 2x^{\frac{3}{2}} + \frac{(x^2+1)\cdot 1}{2\sqrt{x}}$

or :- طريقة الثانية طريقة الجمع المباشر. -:

$y = x^{\frac{1}{2}} \cdot x^2 + x^{\frac{1}{2}} + 1$

$y = x^{\frac{5}{2}} + x^{\frac{1}{2}}$

$y' = \frac{5}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{-1}{2}} = \frac{5}{2}x^{\frac{3}{2}} + \frac{1}{2\sqrt{x}}$

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1) $y = (3x^3 - 1)^3$ power rule. (chain rule).

$y' = \underline{3}(3x^3 - 1)^2 \cdot (\underline{9x^2})$

$y' = 27x^2 \cdot (3x^3 - 1)$

$y' = 27x^2 (3x^3 - 1)^2$

5) $y = \frac{1}{\sqrt[3]{x + 3x^2}} = \frac{1}{(x + 3x^2)^{\frac{1}{3}}} = (x + 3x^2)^{\frac{-1}{3}}$

$\frac{-1}{3} - \frac{3}{3} = \boxed{\frac{-4}{3}}$

$y' = \frac{-1}{3} (x + 3x^2)^{\frac{-4}{3}} \cdot (1 + 6x)$

$y' = \frac{-(1 \cdot (1 + 6x))}{3\sqrt[3]{(x + 3x^2)^4}}$

$\frac{-3}{2} - \frac{2}{2} = \frac{-5}{2}$

6). $y = \frac{1}{\sqrt{(x^2+1)^3}} = \frac{1}{(x^2+1)^{\frac{3}{2}}} = (x^2+1)^{-\frac{3}{2}}$

$y' = \frac{-3}{2} (x^2+1)^{\frac{-5}{2}} \cdot (2x)$

$y' = \frac{-3 \cdot (\cancel{2}x)}{\cancel{2}\sqrt{(x^2+1)^5}} = \frac{-3x}{\sqrt{(x^2+1)^5}}$

Yumna Marei 1200563 — Section 9.7 — مراجعة

Using Derivative Formulas

Rule :-

1) if $y = x^n$ than $y' = n x^{n-1}$

2) if $y = U(x) . V(x)$ than $y' = U(x) . V'(x) + V(x) . U'(x)$

3) if $y = \frac{U(x)}{V(x)}$ than $y' = \frac{V(x) . U'(x) - U(x) . V'(x)}{(V(x))^2}$

4) if $y = (u(x))^n$ = than $y' = n (u(x))^{n-1} . u'(x)$

Example :- Find the derivative of y :-

1) $y = (x^5 + 2x)^3$

$y' = 3(x^5 + 2x)^2 . (5x^4 + 2)$

عشان الضرب

2) $y = (x^2 . (x+1))^3$

$y' = 3(x^2 . (x+1))^2 . (x^2 . 1 + (x+1) . 2x)$

$y' = 3(x^2 . (x+1))^2 . (x^2 + 2x^2 + 2x) = 3(x^2 . (x+1))^2 . (3x^2 + 2x)$

or :- $y = (x^3 + x^2)^3$

$y' = 3(x^3 + x^2) . (3x^2 + 2x)$

Yumna Marei 1200563

Example:-

3). $y = \left(\frac{x^2}{x-1}\right)^5$

$y' = 5\left(\frac{x^2}{x-1}\right)^4 \cdot \left(\frac{((x-1)\cdot 2x) - (x^2 \cdot 1)}{(x-1)^2}\right)$

$y' = 5\left(\frac{x^2}{x-1}\right)^4 \cdot \left(\frac{2x^2 - 2x - x^2}{(x-1)^2}\right)$

$y' = 5\left(\frac{x^2}{x-1}\right)^4 \cdot \left(\frac{x^2 - 2x}{(x-1)^2}\right)$

4). $y = \frac{(x-1)^2}{x+3}$ Cahin Rule. $\Rightarrow$ (Quotient and chain ruale).

Chain Rule.

$y' = \frac{(x+3)\cdot 2(x-1)\cdot 1 - (x-1)^2 \cdot 1}{(x+3)^2}$

$y' = \frac{2(x+3)(x-1) - (x-1)^2}{(x+3)^2}$

5). $y = (x^2-1)\sqrt{3-x^2}$

$y = (x^2-1)(3-x^2)^{\frac{1}{2}}$

$y' = (x^2-1)\cdot \frac{1}{\cancel{2}}(3-x^2)^{\frac{-1}{2}} \cdot (-\cancel{2}x) + (3-x^2)^{\frac{1}{2}} \cdot (2x)$

$y' = \frac{-x(x^2-1)}{\sqrt{3-x^2}} + 2x\sqrt{3-x^2}$

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Example :-

if $R(x) = \dfrac{36000000x}{(2x+500)^2}$, Find the Marginal revenue.

$$MR = R'(x) = \frac{(2x+500)^2 \cdot 36000000 - 36000000x \cdot 2(2x+500) \cdot 2}{\left((2x+500)^2\right)^2}$$

$$= \frac{(2x+500)^{2} \cdot 36000000 - 36000000x \cdot 4\cancel{(2x+500)}}{(2x+500)^4}$$ → بنقوم بأخراج عامل مشترك

$$= \frac{\cancel{2x+500}\left((2x+500) \cdot 36000000 - 36000000x \cdot 4\right)}{\cancel{(2x+500)^4}\,3}$$

$$= \frac{(2x+500) \cdot 36000000 - 4(36000000)x}{(2x+500)^3}$$

Find $R'(50) = \dfrac{(2 \cdot 50+500) \cdot 36000000 - 4 \cdot 50(36000000)}{(2 \cdot 50 + 500)^3}$

$R'(50) = \dfrac{200}{3} \approx 66.67$ → بأقرب لأقرب منزلتين عشريتين

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Section 9.8

Higher - order Derivatives :- مشتقات عليا

* if $y = f(x)$, then $y' = f'(x)$ (the First derivatives)

شكلها الرموز على الشكل التالي :-

$$y' = \frac{dy}{dx} \rightarrow$$ مشتقة الأولى

$$y'' = \frac{d^2y}{dx^2} \rightarrow$$ مشتقة ثانية

* The second derivatives :- $y'' = \frac{d^2y}{dx^2}$.

Example :- Find the second derivaties :-

1) $y = x^4 - 3x^2 + x^{-2}$

$$y' = \frac{dy}{dx} = 4x^3 - 6x - 2x^{-3}$$

$$y'' = \frac{d^2y}{dx^2} = 12x^2 - 6 + 6x^{-4}$$ ⇒ the secound derivatives.

Example :- Find $\frac{d^2y}{dx^2}$ of $y = \sqrt{3x - 2}$ ⇒ $y'' = ??$

$$y = (3x - 2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(3x-2)^{-\frac{1}{2}} \cdot (3)$$

$$y' = \frac{3}{2}(3x-2)^{-\frac{1}{2}}$$

$$y'' = \frac{3}{2} \cdot \frac{-1}{2}(3x-2)^{-\frac{3}{2}} \cdot (3)$$

$$y'' = \frac{-9}{4\sqrt{(3x-2)^3}}$$

$y' = \frac{dy}{dx}$

$y'' = \frac{d^2y}{dx^2}$

$y''' = \frac{d^3y}{dx^3} \Rightarrow y^{(3)}$

$y^{(4)} = \frac{d^4y}{dx^4} \Rightarrow y^{(4)}$

Example:-

Find $f^{(4)}(x)$ if $f(x) = \sqrt{x}$

$f(x) = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2} \cdot \frac{-1}{2} x^{\frac{-3}{2}} = \frac{-1}{4} x^{\frac{-3}{2}}$

$\frac{-3}{2} - \frac{2}{2} = \frac{-5}{2}$

$f'''(x) = \frac{-1}{4} \cdot \frac{-3}{2} x^{\frac{-5}{2}} = \frac{3}{8} x^{\frac{-5}{2}}$

$f''''(x) = \frac{3}{8} \cdot \frac{-5}{2} x^{\frac{-7}{2}} = \frac{-15}{16} x^{\frac{-7}{2}}$

$= \frac{-15}{16\sqrt{x^7}}$

Example:-

- Find the rate of change of $C(x) = x^2 - 3x^3$

$C'(x) = 2x - 9x^2 \Rightarrow C'(x) = \overline{MC}$

مشتقة ثانية.

- Find the rate of change of marginal cost if $C(x) = x^2 - 3x^3 \Rightarrow (MC)'$

$C(x) = x^2 - 3x^3$

$C'(x) = MC = 2x - 9x^2$

$C''(x) = (MC)' = 2 - 18x$

مشتقة السابقة

- E:- $f(x) = x^2 + 2$ Find the rate of chang of $f'(x)$:-

$f(x) = x^2 + 2$

$f'(x) = 2x$

$f''(x) = 2 \rightarrow$. وهو المطلوب

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Section 9.9.

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Applications: Marginals and Derivatives.

Review:

- if $R(x)$ is the total Revenue, than the marginal revenue $= MR = R'(x)$
- if $C(x)$ is the total cost, than the marginal cost $= \overline{MC} = C'(x)$
- if $P(x)$ the total profit function, than the marginal profit $= MP = P'(x)$.

Example:-

Suppose that the demand for local cable TV service is given by $p = 80 - 0.4x$, where P the monthly price per dulres and X is the number of subscribers (in hundresds).

a). Find the Revenue function:-

$R(x) = p \cdot x$

$R(x) = (80 - 0.4x) \cdot x = 80x - 0.4x^2$

b) Find the mangeral revenue at $x = 10$

$MR = R'(x) = 80 - 0.8x$ تقريباً

$R'(10) = 80 - 0.8(10) = 80 - 8 = 72$ (approximatly)

$R(11) - R(10) = 831.6 - 760 = 71.6$ (exact) → بضبط.

* حسب ما بحدد السؤال إذا مش طالب (exact) بحل على (approximatly)

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Example:- Q17 → page 625

$C(x) = 40 + x^2$

a). Find the marginal cost at $x = 5$ and tell what this predicts about the cost of producing 1 additional unite ?

$MC = C'(x) = 2X$

$C'(5) = 2 \cdot 5 = 10 \rightarrow$ تقريبا

the cost of producing one additional unite is about 10 \$.

b). $C(6) - C(5) = ?$

$(40 - 6^2) - (40 - 5^2)$

$(40 - 36) - (40 - 25) = 11\$ \rightarrow$ (exact)

the cost of producing one additional unite is 11 \$.

$C(6) - C(5) \cong C'(5)$.

$11 \approx 10$

Q 36 → page 626.

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→ = slop = m

Cost per unite = $60 + 0.2X$

Selling price = 220$ Equilbiumprice: determined by competitive market, How many unite Shound the firm produce and sell each week to maximize its profit?

$m = 60 + 0.2X$

$p = 220\$$

$C(X) = mX + b$

$C(X) = (60 + 0.2X)X + 0$

$C(X) = 60X + 0.2X^2$

$R(X) = p \cdot X$

$R(X) = 220X$

$P(X) = R(X) - C(X)$.

$= 220X - (60X + 0.2X^2)$

$= 160X - 0.2X^2$

mixamize profit at $X = \frac{-b}{2a} = \frac{-160}{2 \cdot -0.2} = \frac{160}{0.4} = 400$ unite.

at $X = 400$ unite → the maximaze profit $= P(400)$

Maximaz profit ⇒ $P(400) = 160(400) - 0.2(400)^2$

$P(400) = \$32000$

Chapter 10 "Application of Derivatives"

Section 10.1

أعلى قيمة وأقل قيمة ← Relative maxima and mimima

(المشتقة الأولى) تزيد والتناقص ← increasing and decreasing → $f'(x)$

$f'(x) \rightarrow$ Critical values — نقطة الحرجة
Maximum / mimimum — أعلى قيمة / أقل قيمة
increasing / decreasing. — تزايد وتناقص
HPI (Horizontal point of inflection).

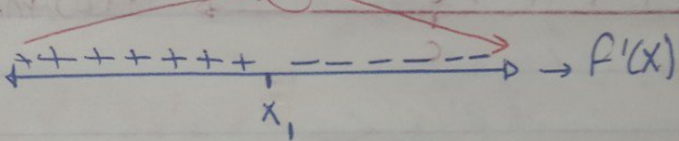
جميع هذه المصطلحات لهم علاقة بالمشتقة الأولى، هكذا السؤال يطلب عندما يريد المشتقة الأولى

* Relative maxima and minima

صورة لو رسمنا $f(x)$.

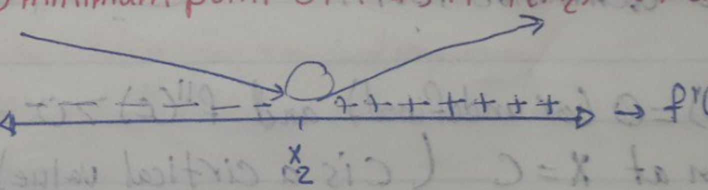
- $(x_1, f(x_1))$ is a relative maximum point of $f(x)$ if $f(x_1) \geq f(x)$ for all x in an interval about x_1

← يوجد قيمة عظمى محلية.



- $(x_2, f(x_2))$ is a relative (local) minimum point of $f(x)$ if $f(x_2) \leq f(x)$ for all x in an interval about x_2.

← يوجد قيمة صغرى محلية.



* Increasing and decreasing.

if $f(x)$ is a function is differentiable on an interval (a,b), than :-

- ① if $f'(x) > 0$ for all x in (a,b), f is increasing on (a,b).

- ② if $f'(x) < 0$ for all x in (a,b), f decreasing on (a,b).

* HPI :-

if $f(x)$ has a relative maximum or a relative ~~min~~ minimum at $x = c$, than $f'(c) = 0$ or $f'(c)$ undefined.

* $f(x)$ has a critical value at C if $f'(c) = 0$ or $f'(c)$ is undefined

$(C, f(C)) \rightarrow$ critcal point.

$\Rightarrow$ if $x = c$ is a critical value for $f(x)$, than $f(x)$ may or may not have a relative maxim or minima at $x = C$.

* first denivative Test (maximum and minimum values)

اختبار المشتقة الأولى :

$f'(x)$: - - - - - 0 + + + + + (at c) → $f(c)$ minimum value.

$f'(x)$: + + + + + + + 0 - - - - - (at c) → $f(c)$ maximum value.

* Second denivative Test :- إختبار المشتقة الثانية

_ if $f'(c) = 0$ (or undefined) and $f''(c) > 0$ than $f(x)$ has a relative minimum at $x = c$ (c is a cirtical value).

_ if $f'(c) = 0$ or (undefined) and $f''(c) < 0$ than $f(x)$ has a relative maximime at $x = c$

$f(x)$

$\downarrow$

$f'(x) = 0$ or undifiend $\rightarrow$ Critcal value

$f'(x)$ ← first derivatov Test

second derivativ Test → $f''(x)$

$f'(x)$

if Given $f(x)$?

Maximum, minimum, increasing, decresing, cirtical values.

1). Find $f'(x)$

2). find the cirtical valuer $\Rightarrow f'(x) = 0$ or $f'(x)$ is undefind.

3). Find sign diagram for $f'(x)$.

$$\begin{array}{ccccc} - - - - - & & + + + + + & & - - - - - \quad f'(x) \\ \hline & a & & b & \end{array}$$

Sign :- $- \longrightarrow +$ $x = a$ minimum $\rightarrow f(a)$.

$+ \longrightarrow -$ $x = b$ mixamum $\rightarrow f(b)$.

$+ \longrightarrow +$ HPI (horizantal point

$- \longrightarrow -$ of inflechion) neither max. nor min.

Example :- let :- $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 8$

1- Find the critical value / 2- Find the relative maximum and minimum /

3). Find the inereasing and decrasing of $f(x)$.

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 8$$

$$f'(x) = \frac{1}{\cancel{4}}\cancel{4}x^3 - \frac{1}{\cancel{3}}.3x^2 - 3.2x$$

$$f'(x) = x^3 - x^2 - 6x$$

1). Critical values $\rightarrow f'(x) = 0$ / $f'(x)$ is undefind $\boxed{X}$ $\rightarrow$ لانه لايوجد مقام

$f'(x) = 0$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x-3)(x+2) = 0$

$x = 0$ / $x = 3$ / $x = -2$

علي

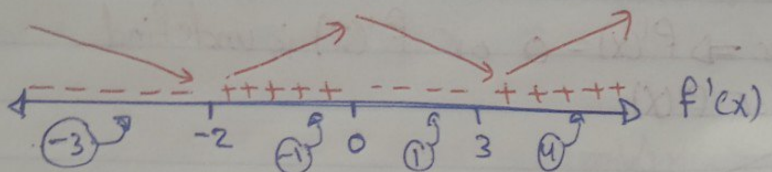
Example :- ⇐

Critical values = $\{0, 3, -2\}$.

Critical point = $(0, f(0)), (3, f(3)) (-2, f(-2))$

$(0, 8), (3, \frac{-31}{4}), (-2, \frac{8}{3})$.

(2)



لإيجاد خط الاشارة نعوض
قيم بين النقاط الحرجة في f'

relative minimum

relative mimimum at $x = -2, 3$ $\left(f(-2) = \frac{8}{3},\ f(3) = \frac{-31}{4}\right)$.

relative maximum at $x = 0$ $(f(0) = 8)$ relative maximum.

(3). $f(x)$ increasing if $f'(x) > 0$

$(-2, 0) \cup (3, \infty)$

$f(x)$ decreasing if $f'(x) < 0$

$(-\infty, -2) \cup (0, 3)$.

Example :- if $y = x^{\frac{1}{3}} \cdot (x-4)$ and $y' = \frac{4(x^{\frac{1}{3}} - 1)}{3x^{\frac{2}{3}}}$

1. Find the circal value / 2. Find the incrasing and dicrasing
3. Find the relative maximumand minume.

(1) cricalvalues = $f'(x) = 0 \rightarrow 0 =$ البسط / $f'(x)$ is undifind $\rightarrow$ المقام = 0

$y' = 4(x-1) = 0 \Rightarrow x = 1$

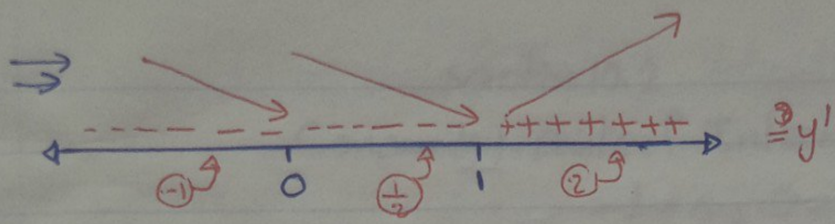
y' undifined $\Rightarrow 3x^{\frac{2}{3}} = 0 \rightarrow x^{\frac{2}{3}} = 0 \Rightarrow x = 0$

Crical values = $\{0, 1\}$.

Crical point = $(0, f(0)), (1, f(1))$

$(0, 0), (1, -3)$

(2)



y-decrasing $(-\infty, 1)$.
y incrasing $(1, \infty)$

يدون الصورة ويعوض في الاقتران الاصلي .

- at $x = 1$ ~~mirimum~~ minimum value $= f(1) = -3$
- no maximum value.
- at $x = 0$ HPI $(0,0)$.

Saction 10.2

Concavity, points of Inflection :- نقطة الانعطاف

* $f(x)$ is concave up (مقعر للأعلى) on an interval I if $f''(x) > 0$ on I

* $f(x)$ is concave down on an interval I if $f''(x) < 0$ on I.

* The point where concavity changes is called an inflection point.

⇒ ① Find $f'(x)$ and $f''(x)$

② $f''(x) = 0$ or $f''(x)$ undifinurd.

③. sing of $f''(x)$. ⟵⟶ $f''(x)$.

if $f''(x) > 0 \rightarrow$ Concave up ∪

if $f''(x) < 0 \rightarrow$ Concave down ∩

if the sign of $f''(x)$ * $+ \rightarrow -$ / $- \rightarrow +$ (inflection point).

Example :- Find the points of inflection and concavity of :-

$y = \frac{1}{2} x^4 - x^3 + 5$

$y' = \frac{1}{2} \cdot 4x^3 - 3x^2 \Rightarrow y' = 2x^3 - 3x^2$

$y'' = 6x^2 - 6x$

$y'' = 0$

$6x^2 - 6x = 0$

$6x(x-1) = 0$

$x = 0 \,/\, x = 1$

⟹

Example :-

② - $6x(x-1)=0 \Rightarrow x=0 \, , \, x=1$

$$\begin{array}{ccccccccc} & + + + + + + + & & - - - - - - & & + + + + + + + & y''(x) \\ -1 & & 0 & \frac{1}{2} & 1 & & 2 & \end{array}$$

$y''=6x^2-6x$

y Concave up $(-\infty,0)\cup(1,\infty)$

y concave down $(0,1)$

inflechion points $(0,f(0))$, $(1,f(1))$

$(0,5)$, $(1,\frac{9}{2})$

Example : (Diminshing Returns) $\rightarrow$ (profit function)

$$P(x) = -0.2x^3 + 3x^2 + 6$$

Find the point of dimnshing return for the profit :-

the point on the graph of $p(x)$ corresponds to the maximum point of the graph of $p'(x)$ and the zero of the graph of $p''(x)$

$x = ??$

$p''(x) = 0 \Rightarrow$ inflection point

$\overleftrightarrow{+++++ \quad | \quad -----}$ $p''(x)$
x

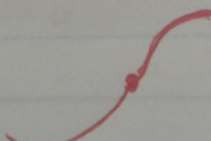
$p'(x) = -0.2x^3 + 3x^2 + 6$

1- $p'(x) = -0.2 \cdot 3x^2 + 3 \cdot 2x$
$p'(x) = -0.6x^2 + 6x$

2. $p''(x) = (-0.6) \cdot 2x + 6$
$p''(x) = -1.2x + 6$
$p''(x) = 0$
$= -1.2x + 6 = 0 \Rightarrow \frac{-1.2x}{-1.2} = \frac{-6}{-1.2}$
$\quad\;\; -6 \quad -6$

$x = 5$. $\overleftrightarrow{++++++ \quad | \quad ------}$ $p''(x)$.
$0 \qquad 5 \qquad \boxed{6}$

$(5, P(5))$ diminshing returns $\rightarrow$ عندما تتغير العنة من موجب إلى سالب
$(5, 56)$. تسمى (diminshing returns) لأنو تتغير من تقعر للأسفل إلى التقعر الأعلى.



ورسمة الربح تكون على الشكل التالي :-

True or Faulse ($f(x)$ defined for all real number)

1- if $f'(c)=0$, than c is a critical value (T)

2- if $f''(c)=0$, than $f(x)$ has a relative maximum or minimum (F) or (H.P.I)

3. if $f(x)$ has a maximum value at c than $f'(c)=0$. (T)

4. if $f'(c)$ is undifined and $f'(x)<0$ to the lefte of c and $f'(x)>0$ to the right of c than $f(x)$ has maximum value at $x=c$. (F)

5. if $f''(a)=0$, than $f(x)$ has inflection point at $(a, f(a))$ (F)

6). if $f''(a)$ is undefind and $f''(x)>0$ to the left of a and $f''(x)<0$ to the right of a than $f(x)$ has inflection point at $x=a$. (T)

7. if $f'(c)=0$ and $f''(c)>0$ than $f(x)$ has relative maximum at c. (F)

Section 10.3

[optimization in Business and Economics.

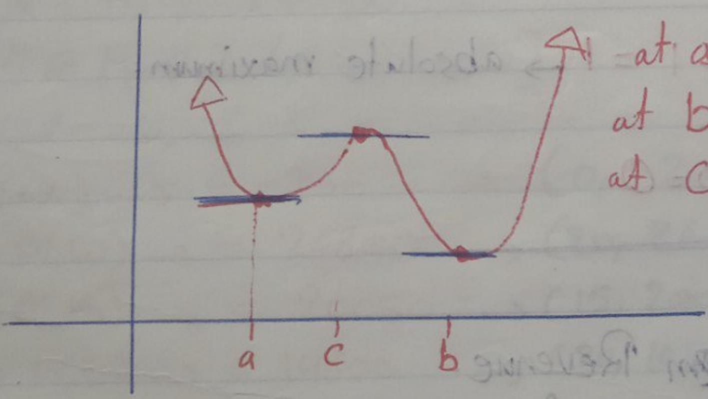
Absolute Extrema

① - The value f(a) is the absolute maximum of f(x) if $f(a) \geq f(x)$ for all x in the Domain of f(x)

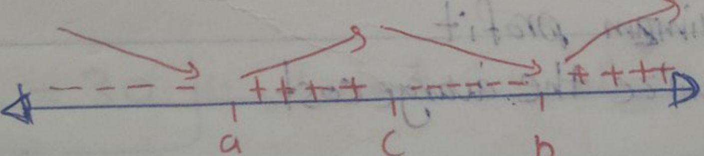
② - The value f(b) is the absolute minimum of f(x) if $f(b) \leq f(x)$ for all x in the Domain of f(x).

* if f(x) is continuos on a clased interval [a,b] than f(x) takes on both an absolute maximum and absolute minimum.

* The absolute maximum or absolute minimum may ocaur at the end points of the interval or at the critical values of f(x).



at a → relative minimum.
at b → absolute minmum.
at c → relative maximum.



F(b) < F(a)

F(b) aboslute minimum
no aboslute maximum.

Example :-

Find the absolute maximum and minimum (on) $[a, b]$. → فترة مغلقة بين a و b

$F(x) = x^3 + x^2 - x$, $[-2, 0]$

$F(x)$ continuosc (polynomait)

Cricale value :-

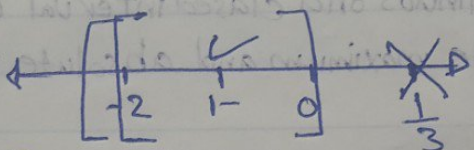
$f'(x) = 3x^2 + 2x - 1$

$f'(x) = 0$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3}$ $\quad x = -1$



$\frac{1}{3} \notin [-2, 0]$ / $-1 \in [-2, 0]$

$-2 \rightarrow f(-2) \Rightarrow -8 + 4 - 2 = -2 \rightarrow$ absolute minimum.

$-1 \rightarrow f(-1) \Rightarrow -1 + 1 + 1 = 1 \rightarrow$ absolute maximum.

$0 \rightarrow f(0) \Rightarrow 0 + 0 - 0 = 0$

$\boxed{\text{optimization}}$ → maximum Revenue.

→ maximum profit

→ minimize the avarge cost

أي كلمة هاي [illegible] الطلب على [illegible]

Example:-

* عدد الوحدات لا تزيد عن 30 وحدة .

Suppose that the production capacity for a certain product can not exceed 30. if the total profit for this product is :-

$[0,30] \leftarrow 0 \leq x \leq 30$ نعتبر الفترة ∆

$P(x) = 4x^3 - 210x^2 + 3600x - 200$

Find the number (x) of items that will maximize profit solution :-

$P'(x) = 4 \cdot 3x^2 - 210 \cdot 2x + 3600$

Critical values

$P'(x) = 0$

$12x^2 - 420x + 3600 = 0$

$12(x^2 - 35x + 300) = 0$

$12(x - 15)(x - 20) = 0$

$x = 15 , \quad x = 20$

$15 \in [0,30] \; / \; 20 \in [0,30]$

$0 \rightarrow P(0) \rightarrow = -200 \rightarrow (0, -200)$

$30 \rightarrow P(30) \rightarrow = 26800 \rightarrow (30, 26800)$ aboslut maximum

$15 \rightarrow P(15) \rightarrow = 20050 \rightarrow (15, 20050)$

$20 \rightarrow P(20) \rightarrow = 19800 \rightarrow (20, 19800)$

$\boxed{x = 30}$ → the number ← المطلوب

Example:-

if the total cost $C(x) = \frac{1}{4}x^2 + 4x + 100$ produaing how many unites will result in a minimum avarge cost.

avarge cost $= \bar{C}(x) = \frac{C(x)}{x}$

$$\bar{C}(x) = \frac{\frac{1}{4}x^2 + 4x + 10}{x} = \frac{1x^2}{4x} + \frac{4x}{x} + \frac{100}{x}$$

$$\bar{C}(x) = \frac{1}{4}x + 4 + 100x^{-1}$$

$$(\bar{C}(x))' = \frac{1}{4} + -1 \times 100\, x^{-2}$$

$$\frac{x^2 \cdot 1}{x^2 \cdot 4} + \frac{-100 \cdot 4}{x^2 \cdot 4} = \frac{x^2 - 400}{4x^2}$$

Cricale values

$(\bar{C}(x))' = 0 \Rightarrow x^2 - 400 = 0 \Rightarrow \sqrt{x^2} = \sqrt{400} = x = 20$, $x = -20$ X مرفوضة لانو لايوجد عدد وحدات بالسالب

$(\bar{C}(x))'$ undifiend $\to 4x^2 = 0 \to x = 0 \to$ مرفوضة

Critical valuse $= 20$

First derivative Test

- - - - 20 + + + + $(\bar{C}(x))'$

minimum value at $x = 20$

secound derivat Test

$$(\bar{C}(x))'' = 200x^{-3} = \frac{200}{x^3}$$

$$(\bar{C}(x))''(20) = \frac{200}{(20)^3} > 0$$ minimum at $x = 20$

Section 11.1

Derivative of logarithmic Functions

For $a > 0$, $a \neq 1$, the Logarithimic Functions.

$$y = \log_a x \iff a^y = x \quad , \quad x > 0 \text{ (Domain)}.$$

$$\log_e x = \ln x \quad / \quad \log_{10} x = \log x.$$

Rules :-

1. $\log_a a^x = x$
2. $a^{\log_a x} = x$
3. $\log_a MN = \log_a M + \log_a N$
4. $\log_a \frac{M}{N} = \log_a M - \log_a N$
5. $\log_a M^r = r \log_a M$.
6. $\log_a 1 = 0$

1. $\ln e^x = x$
2. $e^{\ln x} = x$
3. $\ln MN = \ln M + \ln N$
4. $\ln \frac{M}{N} = \ln M - \ln N$.
5. $\ln M^r = r \ln M$
6. $\ln 1 = 0$

Rule :-

1. Derivative of $y = \ln x$

   if $y = \ln x$ than $\frac{dy}{dx} = y' = \frac{1}{x}$

2. Derivative of $y = \ln u$. $u(x)$

   if $y = \ln u$ than $\frac{dy}{dx} = y' = \frac{1}{u} \cdot u' = \frac{u'}{u}$

Example :- $y = \ln x^3 \Rightarrow y' = \frac{3x^2}{x^3}$

③. Derivative of $y = \log_a x$

if $y = \log_a x$ than $y' = \dfrac{1}{(\ln a)x}$

$y = \log_a x = \dfrac{\ln x}{\ln a}$

$y' = \dfrac{1}{\ln a} \cdot \dfrac{1}{x}$ مشتقة

دائماً موجود في مشتقة ال $\boxed{\log}$ ثابت.

④. Derivative of $y = \log_a u$, $u(x)$

عبارة عن اقتران.

if $y = \log_a u$ than $y' = \dfrac{1}{(\ln a)u} \cdot u'$

Examples :-

✻ Find the derivative of the following :-

1- $y = \log_3 x \rightarrow y' = \dfrac{1}{(\ln 3) \cdot x}$

2- $y = \ln x \rightarrow y' = \dfrac{1}{x}$

3- $y = 3\ln(5x) \rightarrow y' = 3 \cdot \dfrac{5}{5x} = \dfrac{\cancel{15}^{3}}{\cancel{5}_{1}x} = \dfrac{3}{x}$

4. $y = 5\ln(x^2 + 3x) \rightarrow y' = 5 \cdot \dfrac{2x+3}{x^2+3x} = \dfrac{10x+15}{x^2+3x}$

5. $y = 2\log_3 (x^{-2} + x^3)$

$y' = 2 \cdot \dfrac{1}{\ln(3)} \cdot \dfrac{-2x^{-3} + 3x^2}{x^{-2} + x^3} = \dfrac{2}{\ln(3)} \cdot \dfrac{-2x^{-3} + 3x^2}{x^{-2} + x^3}$

6. $y = \dfrac{1}{2} \cdot \ln \dfrac{1}{\sqrt{x^2+1}} = \dfrac{1}{2} \cdot \ln (x^2+1)^{-\frac{1}{2}} \Rightarrow \dfrac{1}{2} \cdot \dfrac{-1}{2} \ln (x^2+1)$ $\left(\ln M^r = r \ln M\right)$

$y = \dfrac{-1}{4} \ln(x^2+1) \rightarrow y' = \dfrac{-1}{4} \cdot \dfrac{2x}{x^2+1} = \dfrac{-2x}{4x^2+4}$

Example:-

7). $y = x^3 + 3\ln(7x)$.

$y' = 3x^2 + 3 \cdot \frac{\cancel{7}}{\cancel{7}x} \Rightarrow 3x^2 + \frac{3}{x}$

مشتقه صفر لانو ثابت

8). $y = \ln 3 + 5x$

$y' = 5$

9)- $y = \log_5 (x^3+1)^5 \Rightarrow y = 5\log_5 (x^3+1)$

$\log_a M^r = r \log_a M$.

$y' = 5 \cdot \frac{1}{\ln 5} \cdot \frac{3x^2}{x^3+1} = \frac{5}{\ln 5} \cdot \frac{3x^2}{x^3+1} = \frac{15x^2}{(\ln 5) \cdot (x^3+1)}$

10)- $y = \ln(x(x^2+5)^6)$

$\ln M \cdot N =$
$\ln M + \ln N$

$\ln M^r = r \ln M$.

$y = \ln x + \ln(x^2+5)^6$

$y = \ln x + 6\ln(x^2+5)$

$y' = \frac{1}{x} + 6 \cdot \frac{2x}{x^2+5} = \frac{1}{x} + \frac{12x}{x^2+5}$

11). $y = \log(3x+6)$.

$y' = \frac{1}{\ln 10} \cdot \frac{3}{3x+6}$

Example:-

$$\ln \frac{M}{N} = \ln M - \ln N$$

$$\ln M^r = r \ln M$$

$\boxed{12}$. $y = \ln\left(\dfrac{\sqrt[3]{3x+5}}{x^2+11}\right)^4$

$$y = 4\ln\left(\frac{\sqrt[3]{3x+5}}{x^2+11}\right)$$

$$y = 4\left[\ln(3x+5)^{\frac{1}{3}} - \ln(x^2+11)\right]$$

$$y = 4\left[\frac{1}{3}\ln(3x+5) - \ln(x^2+11)\right]$$

$$y' = 4\left[\frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{3x+5} - \frac{2x}{x^2+11}\right]$$

$$y' = 4\left[\frac{1}{3x+5} - \frac{2x}{x^2+11}\right]$$

$$y' = \frac{4}{3x+5} - \frac{8x}{x^2+11}$$

$\boxed{13}$. $y = \ln 5 - \ln 2x$

$$y' = 0 - \frac{\cancel{2}\,1}{\cancel{2}x} = \frac{-1}{x}$$

Section 11.2
Derivative of Exponential Functions

* Derivative of $y = e^x$

if $y = e^x$, than $\frac{dy}{dx} = y' = e^x$

* If $y = e^u$, $u(x)$

than $y' = e^u \cdot u'$

* If $y = a^x$ than $y' = a^x \cdot \ln a$ $\Rightarrow a =$ رقم موجب

* If $y = a^u$ than $y' = a^u \cdot \ln a \cdot u'$

Example :-

Find the derivative of y :-

1) $y = e^x \longrightarrow y' = e^x$

2) $y = e^{3x} \longrightarrow y' = e^{3x} \cdot 3$

3) $y = e^{5x^2+3} \longrightarrow y' = e^{5x^2+3} \cdot 10x$

4) $y = e^{2x+3x^3} \longrightarrow y' = e^{2x+3x^3} \cdot (2+9x^2)$

5) $y = 4^x \longrightarrow y' = 4^x \cdot \ln 4$

6) $y = 4^{x^2} \longrightarrow y' = 4^{x^2} \cdot \ln 4 \cdot 2x$

7) $y = 5^{x^2+x} \longrightarrow y' = 5^{x^2+x} \cdot \ln 5 \cdot (2x+1)$

8) $y = e^3 \longrightarrow y' = 0$

عدد ثابت
والمشتقة أصفر.

Example :-

$\boxed{9}$. $y = 2e^{\sqrt{x}} \longrightarrow y' = \cancel{2}e^{\sqrt{x}} \cdot \frac{1}{\cancel{2}\sqrt{x}} = \frac{1}{\sqrt{x}} e^{\sqrt{x}}$

$u = \sqrt{x} \rightarrow x^{\frac{1}{2}}$

$u' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\boxed{10}$. $y = \frac{2}{e^{2x}} + xe^{x}$

$y = 2e^{-2x} + \underline{xe^{x}} \longrightarrow$ product Rule :- مشتقة الضرب

$y' = 2e^{-2x} \cdot -2 + (x \cdot e^{x} + e^{x} \cdot 1)$

$y' = -4e^{-2x} + xe^{x} + e^{x}$

$\boxed{11}$ - $y = \frac{x}{1+e^{3x}}$

$y = x \cdot (1+e^{3x})^{-1}$

product + Chain Rule.

$y' = x \cdot -1\,(1+e^{3x})^{-2} \cdot e^{3x} \cdot 3 + (1+e^{3x})^{-1} \cdot 1$

$y' = \frac{-3xe^{3x}}{(1+e^{3x})^{2}} + \frac{1}{1+e^{3x}}$

or :- بالقسمة

$y' = \frac{(1+e^{3x}) \cdot 1 - x \cdot e^{3x} \cdot 3}{(1+e^{3x})^{2}} = \frac{\cancel{1+e^{3x}} - 3xe^{3x}}{(1+e^{3x})^{\cancel{2}}}$

$= \frac{1}{1+e^{3x}} - \frac{3xe^{3x}}{1+e^{3x}}.$

Section 11.3

Implicit Differentiation :- الاشتقاق الضمني

$$y = (x^3 + 2x)^2$$

$$y' = 2(x^3 + 2x)\cdot(3x^2 + 2)$$

هذا الاشتقاق العادي ←

أما الاشتقاق الضمني يختلف عن الاشتقاق العادي ←

$$\frac{dy}{dx} = y' \ ?$$

ملاحظة :- عند اشتقاق y يجب وضع y' لانو هي أساس السؤال وهي المجهول الذي يجب أن أخرجه .

$$* \ y^2 = x$$

$$\frac{dy}{dx} = \frac{2y\cdot y'}{2y} = \frac{1}{2y}$$

$$y' = \frac{1}{2y}\Big|_{(4,2)} = \frac{1}{2.2} = \frac{1}{4}$$

Example :- Slope of Tangent :- مشتقة أولى

$$x^2 + y^2 - 9 = 0 \qquad (\sqrt{5}, 2). \ / \ \text{Fiend } y'(\sqrt{5}, 2).$$

$$2x + 2y\cdot y' - 0 = 0 \qquad y'(\sqrt{5}, 2).$$

$$\cancel{2\cdot(\sqrt{5})} + 2\cdot(2)\,y' = 0$$

$$-2\cdot(\sqrt{5}) \qquad -(2\sqrt{5})$$

$$\frac{4y'}{4} = \frac{-2\sqrt{5}}{4} \Rightarrow y' = \frac{-\sqrt{5}}{2}$$

Example :- $\frac{dy}{dx} = y'$

$$\ln(x\cdot y) = 6$$

$$\ln x + \ln y = 6$$

$$\cancel{\frac{1}{x}} + \frac{1\cdot y'}{y} = 0$$

$$-\frac{1}{x} \qquad -\frac{1}{x}$$

$$\Rightarrow y\cdot\frac{y'}{y} = \frac{-1}{x}\cdot y \Rightarrow y' = \frac{-y}{x}$$

Example :- Demand

$$p = \frac{10000}{(x+1)^2} \qquad \text{Find } \frac{dx}{dp} = \frac{dq}{dp}.$$

$dx = x'$

لكن يتم كتابتها $\frac{dx}{dp}$ ونهمل
p معامل X ← يعني مشتقة 1.

$$p = 10000\,(x+1)^{-2}$$

$$1 = 10000 \cdot -2\,(x+1)^{-3} \cdot 1 \cdot \frac{dx}{dp}$$

ملاحظة :- يجب عند اشتقاق X يجب وضع $x' = \left(\frac{dx}{dp}\right)$
لأنو هي أساس السؤال وهي المجهول الذي
يجب إيجاده.

$$\frac{1}{1} = \frac{-20000}{(x+1)^3} \cdot \frac{dx}{dp}$$

$$\frac{(x+1)^3}{-20000} = \frac{\cancel{-20000}}{\cancel{-20000}} \cdot \frac{dx}{dp}$$

$$\frac{dx}{dp} = \frac{-(x+1)^3}{20000}$$

Example :-

معاملة مشتقة الضرب.

$$x^3 + \underbrace{xy} + y = 0 \qquad \text{Find } \frac{dy}{dx} = y'$$

$$3x^2 + x \cdot y' + y \cdot 1 + 0 = 0$$

$$3x^2 + x \cdot y' + y + 0 = 0$$

$$3x^2 + x \cdot y' + y = 0.$$

Section 11.5

Applications in Business and Economics

Elastiaity of Demand (η or E).

* The Elastiaicity of Demand at point (q, p):

is $\eta = \frac{-p}{q} \cdot \frac{dq}{dp}\Big|_{(q,p)} \to q'$

Example:

Find the elasiticity of the demand function $D: p + 5q = 100$
when 1) $p = 40$ 2) $p = 60$ 3) $p = 50$

$p + 5q = 100$

طريقة الاشتقاق الضمني:

$1 + 5q' = 0$
$\quad -1 \quad -1$

$\frac{5q'}{5} = \frac{-1}{5}$

$q' = \frac{-1}{5}$

طريقة الاشتقاق العادي

$p + 5q = 100$
$-p \qquad -p$

$\frac{5q}{5} = \frac{100}{5} - \frac{p}{5}$

$q = 20 - \frac{1}{5}p$

$q' = 0 - \frac{1}{5}$

$\frac{dq}{dp} = q' = \frac{1}{5}$

$\eta = \frac{-p}{q} \cdot \frac{dq}{dp}$

$\eta = \frac{-p}{q} \cdot \frac{+1}{5}$

$\eta = \frac{p}{5q}$

1) $p = 40 \Rightarrow 40 + 5q = 100 \Rightarrow \frac{5q}{5} = \frac{60}{5} \Rightarrow q = 12$
$\quad -40 \qquad -40$

$\eta = \frac{40}{5(12)} = \frac{2}{3} < 1$ inelastic → طلب غير مرن.

2) $p = 60 \Rightarrow 60 + 5q = 100 \Rightarrow \frac{5q}{5} = \frac{40}{5} \Rightarrow q = 8$
$\quad -60 \qquad -60$

$\eta = \frac{60}{5(8)} = \frac{3}{2} > 1$ elastic . طلب مرن

3). $p = 50 \to 50 + 5q = 100 \to \frac{5q}{5} = \frac{50}{5} \to q = 10$
$\quad -50 \qquad -50$

$\eta = \frac{50}{5 \cdot 10} = \frac{50}{50} = 1$ unitary elasctic

* تغير في السعر يؤدي إلى التغير في الطلب.

$$\eta \begin{cases} \eta > 1 \text{ elastic} & \text{مرن} \\ \eta < 1 \text{ inelastic} & \text{غير مرن} \\ \eta = 1 \text{ unitary elastic.} \end{cases}$$

Note:

* if $\eta > 1$, the demand is elastic (the precent decrease in demand is greater than the corresponding percent increase in price).

وتنخفض الكمية q ... تغير كبير ... يزيد السعر P

* if $\eta < 1$ the demand is inelastic (the precent decrease in demand is less than the corresponding percent increase in price.).

وتتغير صغير.

* if $\eta = 1$ the demand is unitary elastic (the percent decrease in demand is approximately equal to the corresponding percent increasine in price).

تتغير بشكل تقريبي نفس بعض.

Example :-

The demand function given by $P = \frac{1000}{(q+1)^2}$ Find the elasticity of demand with respect to price when $q = 19$.

$\eta = \frac{-P}{q} \cdot \frac{dq}{dP}$ $\Rightarrow q = 19 \rightarrow P = \frac{1000}{(19+1)^2} = \frac{1000}{400} = 2.5 \Rightarrow (19, 2.5)$

$\frac{dq}{dP} = ??$

$P = 1000 \cdot (q+1)^{-2}$

$1 = 1000 \cdot -2(q+1)^{-3} \cdot q'$

$\frac{1}{-2000(q+1)^{-3}} = \frac{\cancel{-2000(q+1)^{-3}}}{\cancel{-2000(q+1)^{-3}}} \cdot q'$

$q' = \frac{1}{-2000(q+1)^{-3}} \Rightarrow q' = \frac{-(q+1)^3}{2000}$

$\eta = \frac{-P}{q} \cdot \frac{-(q+1)^3}{2000} = \frac{(2.5) \cdot (20)^3}{2000 \cdot 19} = \frac{20000}{38000} \approx 0.53$

$0.53 < 1$ in elastic.

Not :-

Elasticity and Revenae :-

$R(x) = P \cdot x$ → q

1)- Elastic ($\eta > 1$), $\frac{dR}{dP} < 0$ ⇒ if p increases revenue decreases and if p decrases the revenue increases. علاقة عكسية

2)- Inelastic ($\eta < 1$), $\frac{dR}{dP} > 0$ ⇒ if p increases, revenue increases and. if p decrease ~~and~~ revenue decrease. علاقة طردية

3) unitary elastic ($\eta = 1$), $\frac{dR}{dP} = 0$ ⇒ an increase or decrease in price will not Change revenue → (mixmum revenue) → $p \cdot q$

Example:-

if the demand function given by $D: q = 400 - p^2$

1) Find the elasticity of demand when $p = 15$.

2) How an increase in price will affect revenue?

1) $\eta = \frac{-p}{q} \cdot \frac{dq}{dp}$

when $p = 15$

$q = 400 - (15)^2$

$q = 400 - 225$

$q = 175$

$(175, 15)$

$\frac{dq}{dp} = ??$

$q = 400 - p^2$

$q' = -2p \Big|_{p=15}$

$q' = -2(15)$

$q' = -30$

$\eta = \frac{-p}{q} \cdot \frac{dq}{dp}$

$\frac{-15}{175} \cdot -30 = \frac{450}{175} = 2.57 > 1$ elastic.

2) elastic, $\frac{dR}{dp} < 0 \Rightarrow$ if price incrase the total revenue will decrease.

* Revenue :- $q \downarrow \xrightarrow{D} p \uparrow$

$p \uparrow \xrightarrow{\text{elastic}} \downarrow R$

$p \uparrow \xrightarrow{\text{inelastic}} \uparrow R$

Examples:

The demand for a product given $0 \leq q \leq 100$ by $q = 100 - \frac{P^2}{100}$, Find

1) the point at which damand is of unitary elasticity, and Find intervals in which the demand is inelastic and in which elastic.

2) Find q where revenue is increasing, decreasing and where it is maximized.

$$q = 100 - \frac{P^2}{100} \quad \Rightarrow \eta = \frac{-P}{q} \cdot \frac{dq}{dp}$$

$$q' = \frac{1}{100} \cdot 2P \quad \Rightarrow q' = \frac{-1}{50} P$$

$$\underset{-100}{q} = \cancel{100} - \frac{P^2}{100} \quad \Rightarrow \frac{q - 100}{1} = \frac{-P^2}{100}$$

$$P^2 = -100q + 10000 \Rightarrow$$

$$\eta = \frac{-P}{q} \cdot \frac{-P}{50} = \frac{P^2}{50q}$$

$$\eta = \frac{-100q + 10000}{50q}$$

$$\eta = \frac{-2q + 200}{q}$$

unitary elasiticty $\Rightarrow \eta = 1 \Rightarrow \frac{200 - 2q}{q} = \frac{1}{1}$

$$200 - 2q = q$$
$$+2q \quad +2q$$

$$\frac{200}{3} = \frac{3q}{3}$$

$q \approx 66.67$ units

يتبع

Example :- ← تكملة السابق :

مرن

$\eta > 1$ (elastic) (unitary elastic) $\eta = 1$ $\eta < 1$ (inelastic)

0 $q = 1$ $\frac{200}{3} = 66.67$. 100 q

* بعوض في η حتى أعرف أين المرن والغير مرن على قيمة q.

$\eta = \left| \frac{200 - 2.(1)}{(1)} \right| = 198 > 1$ elastic

$(0, \frac{200}{3}) \rightarrow$ elastic.

$(\frac{200}{3}, 100) \rightarrow$ inelastic.

2) Revenue increase if $q \in (0, \frac{200}{3})$.

$0 < q < \frac{200}{3}$ q increse $\Rightarrow$ p decrease (elastic)
$\Rightarrow$ R increses

$\frac{200}{3} < q < 100$ q increses $\Rightarrow$ p decrease (inelastic)
$\Rightarrow$ R decrease

if $q = \frac{200}{3} \longrightarrow$ maximum revenue $= p \cdot q$

$(57.74)(66.67) =$

$3849,53$ \$

$\frac{66.67}{-100} = \frac{100}{-100} - \frac{p^2}{100}$

$\frac{-33.33}{1} = \frac{-p^2}{100}$

$-\times 3333 = -p^2 \times -$

$\sqrt{p^2} = \sqrt{3333}$

$p = 57.74$

Example:- (مهم ـــــ 1) تنسى رمز $\boxed{E = \eta}$

If the elasticity of the demand function for a product is $E = \frac{2p}{600-p}$ where p is unite price and $0<p<600$

1- is the demand elastic or inealstic at $p = 200$?

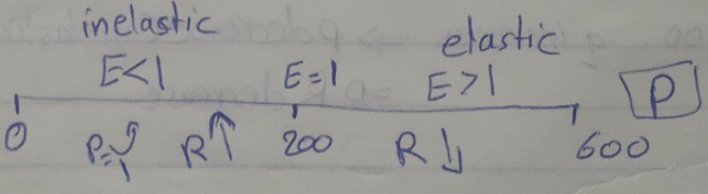
$$E(200) = \frac{2(200)}{600-(200)} = \frac{400}{600-200} = \frac{400}{400} = 1$$ enutiry elastic

2- At $p = 200$, should the price be increased or decrased to incresc the reveinue ?

At $p = 200$, the demand is unit elastic, so the revenue is Maximum, Price should not change.

3- Find the price at which the demand is of unitary elasticity, and Find the intrevals in which the demand is inelastic and in which is elastic.?

$E_d = 1 \quad p = 200$



unite elastic at $p = 200$
elastic $200 < p < 600$
inelastic $0 < p < 200$

section 12.1 Chapter 12

تكامل الغير محدود . Indefinite Intergral

$y = x^2$

$y' = 2x$

$\int 2x\, dx$

$= \frac{\cancel{2}x^2}{\cancel{2}} + C = x^2 + C$ عدد ثابت .

الى داخل تكامل يكون مشتق $\int$

$\int y' = y$

(*) powers of X Formula :—

$$\int n^n\, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

Example:-

1) $\int x^3\, dx = \frac{x^4}{4} + C$

2) $\int x^{\frac{1}{2}}\, dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2x^{\frac{3}{2}}}{3}$

3) $\int \frac{1}{x^2}\, dx \Rightarrow \int x^{-2}\, dx = \frac{x^{-1}}{-1} + C = \frac{-1}{x^1} + C$

*Rules:

1) $\int 1\, dx \Rightarrow \int dx = x + C$

2) $\int C.u(x)\, dx = C \int u.(x)\, dx \Rightarrow \int 3x^2\, dx = 3\int x^2 dx = \cancel{3}\frac{x^3}{\cancel{3}} + C = x^3 + C.$

3) $\int (u(x) \mp v(x))\, dx = \int u(x)\, dx \mp \int v(x)\, dx$.

Examples :-

1) $\int 5\,dx \Rightarrow 5\int dx = 5x + c$

2) $\int 6x^4\,dx \Rightarrow 6\int x^4\,dx = \frac{6x^5}{5} + c$

3) $\int x^3 + 4x + 1)\,dx = \int x^3\,dx + \int 4x\,dx + \int 1\,dx$

$\frac{x^4}{4} + \frac{4x^2}{2} + x + c$

$\frac{x^4}{4} + 2x^2 + x + c.$

4) $\int (x^2 - 2)^2\,dx$

$\int (x^2)^2 - 2.2.x^2 + (2)^2\,dx$

$\int x^4 - 4x^2 + 4\,dx = \frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$

5) $\int \frac{x+1}{x^4}\,dx = \int \frac{x}{x^4} + \frac{1}{x^4}\,dx$

$\int x^{-3} + x^{-4}\,dx = \frac{x^{-2}}{-2} + \frac{x^{-3}}{-3} + c$

$= \frac{-1}{2x^2} + \frac{-1}{3x^3} + c$

6) $\int \frac{2}{5\sqrt[3]{x}}\,dx \Rightarrow \frac{2}{5}\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{2}{5}\int x^{\frac{-1}{3}}\,dx$

$= \frac{2}{5}\,\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{2}{5}\cdot\frac{3}{2}\,x^{\frac{2}{3}} + c$

$\frac{\cancel{6}\,3}{\cancel{10}\,5}\,x^{\frac{2}{3}} + c$

بأبسط صورة . $= \frac{3}{5}x^{\frac{2}{3}} + c$

Not:-

The function $F(x)$ is called an antiderivative of $f(x)$ if $F'(x) = f(x)$
The set of all antiderivative, $F(x) + c$, is called the indefinite integral of $f(x)$ with respect to x and is denoted by :-

$$\int f(x)\,dx = F(x) + c$$

$f(x) \rightsquigarrow F(x)$ / $g(x) \rightsquigarrow G(x)$

Ture or Feals :-

if $x^2 + 3x$ is an antiderivative of $2x + 3$. (True).

$$\int 2x+3\,dx = \frac{\cancel{2}x^2}{\cancel{2}} + 3x + c \Rightarrow x^2 + 3x + c$$ حيث ج أي عدد ثابت

$$\int f(x)\,dx = F(x) + c$$

* $R(x) = 3x^2 + 2x + 1$
  $MR = 6x + 2.$

$MR = 6x + 2 \longrightarrow R(x) = ??$

$R(x) = \int MR\,dx$

$$\int 6x + 2\,dx = \frac{\cancel{6}^{3}x^2}{\cancel{2}} + 2x + c$$

$$= 3x^2 + 2x + c$$ (عدد ثابت)

Example :- في الرباح عندما لا أبيع ولا وحدة فبتالي لا يكون لدي ربح ⇐ $R(0) = 0$

$$MR = R'(X) = 300 - 0.2X \qquad R(X) = ??$$

$$R(X) = \int MR \, dX$$

$$= \int 300 - 0.2X \, dX = 300X - \frac{\overset{0.1}{\cancel{0.2}} X^2}{\cancel{2}} + C$$

$$R(X) = 300X - 0.1X^2 + C \qquad R(0) = 0$$

$$R(0) = 300(0) - 0.1(0)^2 + C$$

$$0 = 0 - 0 + C$$

$$C = 0$$

$$R(X) = 300X - 0.1X^2$$

Total Revenou($X = 1000$)

$$R(1000) = 300(1000) - 0.1(1000)^2$$

$$R(1000) = 300000 - 100000$$

$$R(1000) = 200000.$$

2) if $\int f(x)\,dx = 2x^9 - 7x^5 + C$, Find $f(x)$.

$$f(x) = (2x^9 - 7x^5 + C)'$$

$$f(x) = 18x^8 - 35x^4$$

section 12.2

The power Rule :- قواعد التكامل

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$$

* The power Rule :-

اقتران ، مشتقة الاقتران

$$\int [u(x)]^n \cdot u'(x) \, dx = \frac{[u(x)]^{n+1}}{n+1} + C \quad , n \neq -1$$

Example :- Find the solution :-

$u(x) \cdot u'(x)$

1)- $\int (3x^2+4)^5 \cdot 6x \, dx$ تم تحقق الشرط ! ! $\quad u = 3x^2+4$, $u' = 6x$

$$= \frac{(3x^2+4)^{5+1}}{5+1} + C$$

$$= \frac{(3x^2+4)^6}{6} + C$$

2). $\int \sqrt{2x+3} \cdot 2 \, dx$ $\quad u = 2x+3$, $u' = 2$ (ب)

$$\int (2x+3)^{\frac{1}{2}} \cdot 2 \, dx$$

$$= \frac{(2x+3)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C$$

$$\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{(2x+3)^3}}{3} + C$$

3) $\int x^3 (5x^4+11)^9 \, dx$

$u = 5x^4 + 11$

$u' = 20x^3$

$\frac{1}{20} \int 20x^3 (5x^4+11)^9 \, dx$

$= \frac{1}{20} \frac{(5x^4+11)^{9+1}}{9+1} + C$

$\frac{1}{20} \cdot \frac{(5x^4+11)^{10}}{10} + C = \frac{1}{200} (5x^4+11)^{10} + C$

4) $\int e^{5x} (e^{5x}+3)^4 \, dx$

$u = e^{5x}$

$u' = 5e^{5x}$

$\frac{1}{5} \int 5e^{5x} (e^{5x}+3)^4 \, dx$

$= \frac{1}{5} \cdot \frac{(e^{5x}+3)^5}{5} + C = \frac{1}{25} (e^{5x}+3)^5 + C$

5) $\int \frac{x^2 \cancel{+} 1}{(x^3-3x)^3} \, dx$

$u = x^3 - 3x$

$u' = 3x^2 - 3$

$u' = 3(x^2-1)$

$\int (x^2-1) \cdot (x^3-3x)^{-3} \, dx$

$\frac{1}{3} \int 3(x^2-1) \cdot (x^3-3x)^{-3} \, dx$

$= \frac{1}{3} \cdot \frac{(x^3-3x)^{-3+1}}{-3+1} + C$

$\frac{1}{3} \cdot \frac{(x^3-3x)^{-2}}{-2} = \frac{-1}{6} \cdot \frac{1}{(x^3-3x)^2} + C$

$= \frac{-1}{6(x^3-3x)^2} + C$

6)

$$* \int (x^2+4)^2 \, dx$$

$u = x^2 + 4$

$u' = \underline{\underline{2x}} \rightarrow$ غيرموجودة داخل التكامل

$$\int (x^2)^2 + 2 \cdot 4 \cdot x^2 + (4)^2 \, dx$$

نكامل هنا المثال عادي على طريقة (12.1) بوزع القوس :

$$\int x^4 + 8x^2 + 16 \, dx$$

$$= \frac{x^5}{5} + \frac{8x^3}{3} + 16x + C$$

Example :- (مهم)

Suppose $MR = \frac{600}{\sqrt{3x+1}} + 2$ Find total Revenue.

$$R'(x) = MR \Rightarrow \int MR = R(x)$$

$$R(x) = \int MR = \int \frac{600}{\sqrt{3x+1}} + 2 \, dx$$

$$R(x) = \int 600 (3x+1)^{-\frac{1}{2}} dx + \int 2 \, dx$$

$u = 3x + 1$

$u' = \underline{\underline{3}}$

$$R(x) = \frac{600}{3} \int 3 (3x+1)^{-\frac{1}{2}} dx + \int 2 \, dx .$$

$$R(x) = 200 \cdot \frac{(3x+1)^{\frac{1}{2}}}{\frac{1}{2}} + 2x + C$$

$$R(x) = 400 \cdot \sqrt{3x+1} + 2x + C$$

$C = ??$

$R(0) = 0$

$R(0) = 400 \cdot \sqrt{3 \cdot 0 + 1} + 2 \cdot 0 + C$

$0 = 400 \cdot \sqrt{1} + C$

$0 = 400 + C$

$-400 \quad -400$

$C = -400$

$$R(x) = 400\sqrt{3x+1} + 2x - 400 .$$

$R(20) =$ بعوض مباشرة .

Section 12.3

Integrals Involving Exponential and logarithime Function

* if u is a function of x

$$\int e^u . u' \, dx = e^u + c$$

In particular $\int e^x \, dx = e^x + c$

* if u is a function of x

$$\int \frac{u'}{u} \, dx = \ln|u| + c$$

In partiaular $\int \frac{1}{x} \, dx = \int x^{-1} \, dx = \ln|x| + c.$

Example:-

1) $\int 5e^x \, dx$

$$5\int e^x \, dx = 5e^x + c$$

2) $\int 2x e^{x^2} \, dx$

$u = \cancel{x}^2 = x^2$

$u' = 2x$

$$= e^{x^2} + c$$

3) $\int \frac{x^2}{e^{x^3}} dx$ $\qquad u = -x^3$, $u' = -3x^2$

$\int x^2 e^{-x^3} dx$

$\frac{1}{-3}\int -3x^2 \cdot e^{-x^3} dx$

$= \frac{-1}{3} \cdot e^{-x^3} + C$

4) $\int \frac{3}{e^{2x}} dx$ $\qquad u = -2x$, $u' = -2$

$\int 3 e^{-2x} dx$

$\frac{3}{-2}\int -2 e^{-2x} dx$

$= \frac{-3}{2} e^{-2x} + C = \frac{-3}{2e^{2x}} + C$

5) $\int \frac{4}{4x+8} dx$ $\qquad u = 4x+8$, $u' = 4$

$4 \rightarrow 0$ درجة البسط

$4x+8 \rightarrow 1$ درجة المقام

$= \ln|4x+8| + C$ $\qquad \int \frac{u'}{u} = \frac{4}{4x+8}$ (✓)

6) $\frac{1}{2}\int \frac{2(x-3)}{x^2-6x+1} dx$ $\qquad 2(x-3) \rightarrow (1)$, $x^2-6x+1 \rightarrow (2)$

$u = x^2-6x+1$

$u' = 2x-6$

$2(x-3)$

$\frac{1}{2}\int \frac{2(x-3)}{x^2-6x+1} dx$

$= \frac{1}{2} \ln|x^2-6x+1| + C.$

(7) $\displaystyle\int \frac{x^4 - 2x^3 + 4x^2 - 7x - 1}{x^2 - 2x}\, dx.$

* عندما تكون درجة البسط أعلى من درجة المقام نحل على طريقة القسمة الطويلة وحتى لو كان درجتين متساويات.

$$\begin{array}{r|l} & x^2 + 4 \\ \hline x^2 - 2x & x^4 - 2x^3 + 4x^2 - 7x - 1 \\ & x^4 - 2x^3 \\ \hline & 4x^2 - 7x - 1 \\ & -4x^2 - 8x + \\ \hline & (x - 1) \end{array}$$

$$\int (x^2 + 4) + \frac{x-1}{x^2 - 2x}\, dx.$$

$$\int x^2 + 4\, dx + \frac{1}{2}\int \frac{(2)(x-1)}{x^2 - 2x}\, dx$$

$$u = x^2 - 2x$$
$$u' = 2x - 2$$
$$2(x-1).$$

$$= \frac{x^3}{3} + 4x + \frac{1}{2}\int \frac{2(x-1)}{x^2 - 2x}\, dx$$

$$= \frac{x^3}{3} + 4x + \frac{1}{2}\ln|x^2 - 2x| + c$$

Sectione 12.4.
Application of the Indefinite integral
in Business and Economics.

1). $\overline{MC} = C'(x)$

$\Rightarrow \int \overline{MC}\, dx = C(x)$

Example:. if.

$C(0) = FC$

$\overline{MC} = 3(2x+25)^{\frac{1}{2}}$ if the Fixed cost for the month are \$ 11125, what would be the total cost of producing 300 items per month ?

Fixed cost = 111.25.
$\Rightarrow C(0) = 11125$
$C(300) = ??$

$C(x) = \int \overline{MC}\, dx.$

$C(x) = \int 3(2x+25)^{\frac{1}{2}}\, dx$

$u = 2x + 25$
$u' = 2.$

$C(x) = 3\int (2x+25)^{\frac{1}{2}}\, dx$

$C(x) = \frac{3}{2}\int 2(2x+25)^{\frac{1}{2}}\, dx.$

$C(x) = \frac{3}{2} \cdot \frac{(2x+25)^{\frac{3}{2}}}{\frac{3}{2}} + C$

$C(x) = \frac{\cancel{6}\left(\sqrt{(2x+25)^3}\right)}{\cancel{6}} + C$

$C(x) = \sqrt{(2x+25)^3} + C$

$C(0) = \sqrt{(2\cdot 0+25)^3} + C$

$11125 = \sqrt{(25)^3} + C$

$11125 = \cancel{125} + C \Rightarrow C = 11000 \Rightarrow C(x) = \sqrt{(2x+25)^3} + 11000$
$-125 \quad \cancel{-125}$

$C(300) = \sqrt{(2\cdot 300+25)^3} + 11000$
$15625 + 11000$

$C(300) = 26625.$

Example:-

(2) $MR = R'(x)$

$\Rightarrow R(x) = \int MR \, dx$ $\qquad R(0) = 0$ (✓)

* if $MR = 200 - 4x$ Find the $R(x)$.

$R(x) = \int MR \, dx$.

$R(x) = \int 200 - 4x \, dx$

$R(x) = 200x - \frac{4x^2}{2} + C$

$R(x) = 200x - 2x^2 + C \quad \Rightarrow R(0) = 0$

$R(0) = 200 \cdot (0) - 2 \cdot (0)^2 + C$

$0 = 0 - 0 + C$

$C = 0$ (✓)

$R(x) = 200x - 2x^2$ (✓)

③ Maximium profit :-

① vertex $P(x) = ax^2 + bx + c$

② $P'(x) = 0$

Example :-

$\rightarrow R(0) = 0$

$MR = 400 - 30X$

$MC = 20X + 50$

$\rightarrow C(5) = 1750$

→ رتبة livel (X) ⇒ maximumprofit, optimal level → $MR = MC$

and the total cost of produing 5 units is 1750 at what level should the firm hold production in order to maximumc the profits ?

maximumprofit if $MR = MC$

$\Rightarrow 400 - \cancel{30X} = 20X + 50$

$\quad\;\; \cancel{+30X} \qquad \cancel{+30X}$

$400 = 50X + \cancel{50}$

$-50 \qquad\quad \cancel{-50}$

$\frac{50X}{50} = \frac{350}{50} \Rightarrow X = (7) \text{ level}$

at $X = 7$ maximumprofit. what is the maximumprofit ? $P(7)$.

$R(X) = \int MR \, dX = \int$

$R(X) = \int 400 - 30X \, dX$

$R(X) = 400X - \frac{30X^2}{2} + C \Rightarrow 400X - 15X^2 + C$

$R(0) = 400(0) - 15 \cdot (0)^2 + C$

$0 = C \Rightarrow R(X) = 400X - 15X^2$

$$C(x) = \int MC \, dx$$

$$C(x) = \int 20x + 50 \, dx$$

$$C(x) = \frac{20x^2}{2} + 50x + C$$

$$C(x) = 10x^2 + 50x + C$$

$$C(5) = 10(5)^2 + 50 \cdot 5 + C$$

$$1750 = 250 + 250 + C$$

$$1750 = 500 + C$$

$$-500 \quad -500$$

$$C = 1250 \Rightarrow C(x) = 10x^2 + 50x + 1250$$

Fiexed cost = $C(0)$

$C(0) = 1250$

$$P(x) = R(x) - C(x)$$

$$P(x) = 400x - 15x^2 - (10x^2 + 50x + 1250)$$

$$P(x) = 400x - 15x^2 - 10x^2 - 50x - 1250$$

$$P(x) = 350x - 25x^2 - 1250$$

$$x = \frac{-b}{2a} = \frac{-350}{2(-25)} = \boxed{7}$$

$$P(7) = 350(7) - 25(7)^2 - 1250$$

$$P(7) = 2450 - 1225 - 1250$$

$$P(7) = -25 \quad (\text{loss})$$

$$\int_a^b f(x) = F(x) \Rightarrow \text{Example} = \int_0^1 x^3 \, dx = \left.\frac{x^4}{4}\right|_0^1$$

$$\left(\frac{1}{4}\right) - (0) = \frac{1}{4}$$

تم بحمد الله